Optimal Need-Based Financial Aid*

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Abstract

We study the optimal design of student financial aid as a function of parental income. We derive optimal financial aid formulas in a general model. For a simple model version, we derive mild conditions on primitives under which poorer students receive more aid even without distributional concerns. We quantitatively extend this result to an empirical model of selection into college for the United States that comprises multidimensional heterogeneity, endogenous parental transfers, dropout, labor supply in college, and uncertain returns. Optimal financial aid is strongly declining in parental income even without distributional concerns. Equity and efficiency go hand in hand.

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1 Introduction

In all OECD countries, college students benefit from financial support (OECD, 2014). Moreover, with the goal of guaranteeing equality of opportunity, financial aid is typically need-based and targeted specifically to students with low parental income. In the United States, the largest need-based program is the Pell Grant. Federal spending on this program exceeded $30 billion in 2015 and has grown by over 80% during the last 10 years (College Board, 2015).

One justification for student financial aid in the policy debate is that the social returns to college exceed the private returns because the government receives a share of the financial returns through higher tax revenue (Carroll and Erkut, 2009; Baum et al., 2013). This lowers the effective fiscal costs (i.e., net of tax revenue increases) of student financial aid.\footnote{The Congressional Budget Office (CBO), following a request by the Senate Committee on the Budget, recently documented the growth in the fiscal costs of Pell Grant spending (Alsalam, 2013). Dynamic scoring aspects are neglected in this report: the positive fiscal effects through higher tax revenue in the future are not taken into account. Generally, the CBO does consider issues of dynamic scoring: https://www.cbo.gov/publication/50919.}

In this paper, we study the optimal design of financial aid and show that considering dynamic scoring aspects is crucial to assessing the desirability of need-based programs such as the Pell Grant. The reduction in the effective fiscal costs of student financial aid due to dynamic fiscal effects varies along the parental income distribution. We show that effective fiscal costs are increasing in parental income and are therefore lowest for those children that are targeted by the Pell Grant. The policy implication is that need-based financial aid is desirable not only because it promotes intergenerational mobility and equality of opportunity. Need-based financial aid is also desirable from an efficiency point of view because subsidizing the college education of children from weak parental backgrounds is cheaper for society than subsidizing students from "average" parental backgrounds. The usual equity-efficiency trade-off does not apply for need-based financial aid.

To arrive there, we start with a general model without imposing restrictions on the underlying heterogeneity in the population. Further, besides enrollment, labor supply and savings decisions, we consider dropout, labor supply during college and endogenous parental transfers. We derive a simple optimality condition for financial aid that transparently highlights the key trade-offs. At a given level of parental income, optimal financial aid decreases in the share of inframarginal students, which captures the marginal costs. These costs are scaled down by the marginal social welfare weights attached to these students. Optimal financial aid increases in the share of marginal students\footnote{Those students that are at the margin of attending college with respect to financial aid.} and the fiscal externality per marginal student, which jointly capture the marginal benefits of the subsidy. The fiscal externality is the change in lifetime fiscal contributions causal to college attendance.\footnote{On top of that, financial aid is also increasing in the completion elasticity with respect to financial aid and the fiscal externality due to completing college instead of dropping out. This channel, however, turns out to be quantitatively of minor importance.} For the optimality condition, the
specific reason why marginal students change their behavior due to a change in subsidies (e.g., borrowing constraints or preferences) is not important.

Elasticities linking changes in enrollment behavior to changes in financial aid have been estimated in the literature (e.g., by Dynarski (2003) and Castleman and Long (2016)). These papers provide guidance about the average value of this policy elasticity or about its value at a particular parental income level. However, knowledge about how this elasticity varies along the parental income distribution is missing. Knowledge of those parameters for students from different parental income groups, however, is necessary to analyze the welfare effects of need-based financial aid. Further, these elasticities are not deep parameters but do change as policy changes. The main approach of this paper is therefore a structural model of selection into college that allows us to compute this policy elasticity along the parental income distribution and for alternative policies.

As a first step, however, before studying this empirical model, we consider a simple theoretical setting. We reduce the complexity of the problem by focusing on two dimensions of heterogeneity: (i) parental transfers and (ii) returns to college. Further, we simplify the model by making the problem static, shutting down risk, labor supply during college and dropout. We first show that financial aid is decreasing in parental income even in the absence of distributional concerns if the distribution of returns is log concave (which implies a decreasing hazard rate)\(^4\) and if returns and parental income are independently distributed. We then show that these analytical results extend to the empirically more plausible case of a positive association between parental income and child ability.\(^5\)

We then move to our structural life-cycle model, where we account for earnings risk, dropout, labor supply during college and, importantly, we account for crowd-out of parental transfers by explicitly modeling parental decisions to save, consume and provide transfers to their children. Another additional crucial ingredient of the model is heterogeneity in the psychic costs of education because monetary returns can only account for a small part of the observed college attendance patterns (Heckman et al., 2006). Using data from the National Longitudinal Survey of Youth 1979 and 1997, we estimate the parameters of our model via maximum likelihood and provide a detailed discussion of how variation in the data helps us to identify the crucial parameters.

The model successfully replicates quasi-experimental studies. First, it is consistent with estimated elasticities of college attendance and graduation rates with respect to financial aid expansions (Deming and Dynarski, 2009). Second, it is consistent with the causal impact of parental income changes on college graduation rates (Hilger, 2016). Further, our model

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\(^4\)The hazard rate pins down the ratio of marginal over inframarginal students which is also key in this simplified model.

\(^5\)We obtain this clear analytical result if the ability distribution of high parental income children dominates the distribution of low parental income children in the hazard rate order. For a Pareto distribution, e.g., the property of hazard rate dominance always holds in case of first-order stochastic dominance.
yields (marginal) returns to college that are in line with the empirical literature (Card, 1999; Oreopoulos and Petronijevic, 2013; Zimmerman, 2014).

We find that optimal financial aid policies are strongly progressive. In our preferred specification, the level of financial aid drops by more than 69% moving from the 10th percentile of the parental income distribution to the 90th percentile. The strong progressivity result does not rely on the Utilitarian welfare criterion. We show that a social planner that is only interested in maximizing tax revenues would choose an almost equally progressive financial aid schedule.

Second, our estimates suggest that targeted increases in financial aid for students below the 45th percentile of the parental income distribution, are self-financing by increases in future tax revenue; this implies that targeted financial aid expansions could be Pareto-improving free-lunch policies. Both results point out that financial aid policies for students are a rare case in which there is no equity-efficiency trade-off: education policies which lead to a cost-effective distribution of financial aid are also in line with redistributive concerns and social mobility.

One may have expected that efficiency considerations would make a case against need-based financial aid because of the positive correlation between returns to college and parental income. This correlation is indeed positive in our empirical model, and the fiscal externality of the average marginal student with high parental income is higher than for the average marginal student with low parental income. However, this effect is dominated by the fact that at higher parental income levels many more students are inframarginal. We provide a model-based decomposition to assess which features are most important for the parental income gradient. Once we shut down the relation between parental income and ability of the kids (i.e. returns to college) and preferences for college, the gradient is almost flat. This indicates that differences in financial resources directly only seem to play a minor role.

In a last step, we provide several extensions and robustness checks. We show that our progressivity result also holds if we (i) remove borrowing constraints, (ii) choose the merit-based dimension of financial aid optimally, (iii) allow the government to set an optimal Mirrleesian income tax schedule, (iv) model early educational investments and thereby endogenize ability and (v) if the relative wage for college educated labor is determined in general equilibrium.

Our paper contributes to the existing literature in several ways. Stantcheva (2017) characterizes optimal human capital policies in a very general dynamic model with continuous education choices. The main differences with our approach are twofold. First, theoretically, we study a model with discrete education choices as we find this a natural way to study financial aid policies. As we show, the optimality conditions are quite distinct from the continuous case and different elasticities are required to characterize the optimum.\footnote{This resembles the different results in the optimal tax literature along the extensive versus the intensive margin (Diamond, 1980; Saez, 2001, 2002).} Second, the
extensive margin education decision allows us to incorporate a large degree of heterogeneity without making the optimal policy problem intractable. This allows for a modeling approach that is close to the empirical, structural literature.

Bovenberg and Jacobs (2005) consider a static model with a continuous education choice and derive a “siamese twins” result: they find that the optimal marginal education subsidy should be as high as the optimal marginal income tax rate, thereby fully offsetting the distortions from the income tax on the human capital margin.\footnote{Bohacek and Kapicka (2008) derive a similar result as in a dynamic deterministic environment. In Findeisen and Sachs (2016), we focus on history-dependent policies and show how history-dependent labor wedges can be implemented with an income-contingent college loan system. Koeniger and Prat (2017) study optimal history-dependent human capital policies in a dynastic economy where education policies also depend on parental background. Stantcheva (2015) derives education and tax policies in a dynastic model with multi-dimensional heterogeneity, characterizing the relationship between education and bequest policies.} Lawson (2017) uses an elasticity approach to characterize optimal uniform tuition subsidies for all college students.\footnote{Our work is also complementary to Abbott et al. (2018) and Krueger and Ludwig (2013, 2016), who study education policies computationally in very rich overlapping-generations models.} We contribute to this line of research by developing a new framework to analyze how education policies should depend on parents’ resources\footnote{Gelber and Weinzierl (2016) study how tax policies should take into account that the ability of children is linked to parents’ resources and find that the optimal policy is more redistributive towards low-income families.} and also trade off merit-based concerns. Our theoretical characterization of optimal financial aid (and tax policies) allows for a large amount of heterogeneity, and we tightly connect our theory directly to the data by estimating the relevant parameters ourselves. Finally, the paper is also related to many empirical papers, from which we take the evidence to gauge the performance of the estimated model. Those papers are discussed in detail in Section 4.

We progress as follows. In Section 2 we develop the general model and characterize the optimal policies in terms of reduced-form elasticities. In Section 3 we consider a simplified version of the model, which allows us to transparently study mild conditions on primitives under which financial aid is optimally decreasing in parental income. In Section 4 we describe our estimation approach and discuss the relationship to previous empirical work. Section 5 presents optimal financial aid policies, and Section 6 considers the jointly optimal education and tax policies. In Section 7 we discuss further robustness issues. Section 8 concludes.

\section{Optimal Financial Aid Policies}

In this section we characterize optimal (need-based) financial aid policies for college students. Our approach is to work with a general model and characterize the optimal policy in terms of reduced-form elasticities. This formula is general on the one hand and economically intuitive on the other, providing intuition on which objects determine the optimal financial aid.
Before we move on to a fully specified empirical model, we first focus on a simplified framework in Section 3 and derive conditions on primitives that imply that optimal financial aid is indeed need-based, (i.e., that financial aid is decreasing in parental income). Importantly, this result holds in the absence of distributional concerns between students with different parental backgrounds. In Section 4, we estimate the fully specified structural model, which is mainly a specific case of the model analyzed in Section 2.\footnote{In a few minor and rather technical aspects it is, however, more general, as we elaborate below. Incorporating these generalizations in Section 2 already would significantly complicate notation without adding additional economic insights.} We use the estimated model to solve for the optimal financial aid in the United States under a variety of scenarios and assumptions about the social welfare function.

### 2.1 Individual Problem

Individuals start life in year $t = 0$ as high school graduates and are characterized by a vector of characteristics $X \in \chi$ and (permanent) parental income $I \in \mathbb{R}_+$. Life lasts $T$ periods and individuals face the following decisions. At the beginning of the model, they face a binary choice: enrolling in college or not. If individuals decide against enrollment, they directly enter the labor market and make labor-leisure decisions every period. If individuals decide to enroll in college, they also make a labor-leisure decision during college and at the beginning of the year decide to drop out or continue. Individuals graduate after four years. After graduating or dropping out, individuals enter the labor market.

We start by considering labor market decisions of individuals that either are out of college or have chosen to forgo college altogether. This is a standard labor-leisure-savings problem with incomplete markets. Let $V_t^W(\cdot)$ denote the value function of an individual in the labor market in year $t$. Then the recursive problem is given by

$$
V_t^W(X, I, e, a_t, w_t) = \max_{c_t, l_t} U(c_t, l_t) + \beta \mathbb{E} \left[ V_{t+1}^W(X, I, e, a_{t+1}, w_{t+1}) | w_t \right]
$$

subject to the budget constraint

$$
c_t + a_{t+1} = l_t w_t - T(l_t w_t) + a_t (1 + r) + tr_t(X, I, e, a_t, w_t).
$$

The state variables are the initial characteristics $(X, I)$, the education level $e \in \{H, D, G\}$ (high school graduate, college dropout, college graduate), assets $a_t$, and the current wage $w_t$. The variables $(X, I, e)$ are state variables because they may affect parental transfers $tr_t(X, I, e, a_t, w_t)$ and because they may affect the evolution of future wages. The dependence on the education decision then captures the returns to education. The function $T$ captures
the tax-transfer system. Finally, we assume that the utility function is such that there are no income effects on labor supply.\textsuperscript{11}

For the final period \( T \), the value function is simply given by

\[
V^T_{T}(X, I, e, a_T, w_T) = \max_{c_T, l_T} U(c_T, l_T) \text{ s.t. } c_T = l_T w_T - T(l_T w_T) + a_T (1 + r) + tr_T(X, I, e, a_T, w_T).
\]

Having defined these value functions during the working life, we now turn to the value functions of the different education decisions. The value of not enrolling in college (i.e., choosing education level \( H \)) is simply given by

\[
V^H(X, I) = \mathbb{E} \left[ V^W_1(X, I, e = H, a_1 = 0, w_1) \right].
\]

Regarding the realization of uncertainty, the timing is such that individuals directly enter the labor market in period one and draw their first wage \( w_1 \), which is hence only known after the education decision has been made.

Next, we turn to the decisions during college. Besides the question of how much to work and consume while in college, individuals also make the binary decision of dropping out or staying enrolled. Let \( V^E_t(\cdot) \) denote the value function of an agent currently enrolled in college in year \( t = 4 \). We proceed by backward induction and first define the value of continuing college in period \( t = 4 \), hence finishing college. It is given by

\[
V^E_4(X, I, a_4, \varepsilon_4) = \max_{c_4, l_4^E} U^E(c_4, l_4^E; X, I, \varepsilon_4) + \beta \mathbb{E} \left[ V^W_5(X, I, e = G, a_5, w_5) \right]
\]

subject to

\[
c_4 + a_5 = l_4^E \bar{w}(X, I) + \mathcal{G}(I) - \mathcal{F} + tr^E_t(X, I, a_t, \mathcal{G}(I)) + a_4 (1 + r),
\]

where \( \bar{w}(X, I) \) is the wage that students earn if they work during college and \( \mathcal{F} \) is the tuition fee. We denote work in college by \( l_t^E \) to differentiate work in college from work after entering the labor market.\textsuperscript{12} The term \( \mathcal{G}(I) \) is the amount of (need-based) financial aid a student with parental income \( I \) receives,\textsuperscript{13} and \( tr^E_t(X, I, a_t, \mathcal{G}(I)) \) captures parental transfers in year \( t \) for children that are enrolled in college. Importantly, we allow them to be endogenous with respect to the level of financial aid to account for the potential crowding out of paternal

\textsuperscript{11}We discuss the relaxation of this assumption in Section 7.6.

\textsuperscript{12}We assume these earnings are not taxed. In the data, the average earnings of students who work in college are so low that they do not have to pay positive income taxes; in addition, the vast majority of college students does not qualify for welfare/transfer programs.

\textsuperscript{13}In reality, financial aid depends not only on parental income but also on other characteristics of the student. We abstract from that for simplicity in this section but account for this in the estimated model.
transfers through financial aid. We denote flow utility during college by \( U^E(c_t, l^E_t; X, I, \varepsilon_t) \). The dependence on \((X, I)\) captures nonpecuniary aspects of college education which have been found to be important to explain actual college enrollment patterns.\(^{14}\) The term \( \varepsilon_t \) is a stochastic component of these non-pecuniary aspects.

Next we turn to the value of dropping out in period \( t \), which is simply given by

\[
V^D_t(X, I, a_t) = \mathbb{E}[V^W_t(X, I, e = D, a_t, w_t)].
\]

In period 4, an individual drops out if \( V^D_4(X, I, a_4) \geq V^E_4(X, I, a_4, \varepsilon_4) \). Based on that, we can also define the value of being enrolled for all further periods \( t = 1, 2, 3 \):

\[
V^E_t(X, I, a_t, \varepsilon_t) = \max_{c_t, l^E_t} U^E(c_t, l^E_t; X, I, \varepsilon_t) + \beta \mathbb{E}\max[V^E_{t+1}(X, I, a_{t+1}, \varepsilon_{t+1}), V^D_{t+1}(X, I, a_{t+1})]
\]

subject to

\[
c_t + a_{t+1} = l^E_t w(X, I) + G(I) + tr_t(X, I, a_t, G(I)) + a_t (1 + r),
\]

and where \( a_1 = 0 \). And of course more generally, an individual in period \( t \) drops out in period \( t \) if \( V^D_t(X, I, a_t) \geq V^E_t(X, I, a_t, \varepsilon_t) \). Let \( P_t(X, I, G(I)) \) denote the share of individuals of type \((X, I)\) that do not drop out in period \( t \), and let \( P(X, I, G(I)) = \prod_{t=1}^4 P_t(X, I, G(I)) \) be the proportion of all initially enrolled students that graduate. Importantly the model captures the idea that the dropout decision is endogenous with respect to financial aid.

Finally we denote the value of enrolling into college in the first place as

\[
V^E(X, I) = \mathbb{E}[V^E_1(X, I, a_1 = 0, \varepsilon_1)].
\]

An individual enrolls in college if \( V^E(X, I) \geq V^H(X, I) \).

We now move to policy analysis and make one simplifying assumption for the purpose of clarity. We assume that individuals can only drop out after two years in college such that \( P_t(X, I, G(I)) = 1 \) if \( t \neq 3 \). In Appendix A.2, we state the more general version and show that besides more cumbersome notation, the results are basically unchanged.

### 2.2 Fiscal Contributions

We now define the expected net fiscal contributions for different types \((X, I)\) and different education levels as these will be key ingredients for the policy analysis. We start with the net present value (NPV) in net tax revenues of high school graduates of type \((X, I)\):

\(^{14}\)See Cunha et al. (2005), Heckman et al. (2006) or Johnson (2013).
\[\mathcal{NT}_{NPV}^H(X, I) = \sum_{t=1}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \mathbb{E}(T(y_t)|X, I, H),\]

where \(y_t = w_t l_t\) is total earnings in year \(t\). The equivalent for a college graduate is given by

\[\mathcal{NT}_{NPV}^G(X, I) = \sum_{t=5}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \mathbb{E}(T(y_t)|X, I, G) - G(I) \sum_{t=1}^{4} \left( \frac{1}{1 + r} \right)^{t-1},\]

where the costs for financial aid are subtracted from tax revenue. The fiscal contribution of a dropout is given by

\[\mathcal{NT}_{NPV}^D(X, I) = \sum_{t=3}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \mathbb{E}(T(y_t)|X, I, D) - G(I) \sum_{t=1}^{2} \left( \frac{1}{1 + r} \right)^{t-1}.\]

Finally, we define the expected fiscal contribution of an individual that decides to enroll:

\[\mathcal{NT}_{NPV}^E(X, I) = P(X, I, G(I)) \times \mathcal{NT}_{NPV}^G(X, I) + (1 - P(X, I, G(I))) \times \mathcal{NT}_{NPV}^D(X, I).\]

Before we derive optimal education subsidies, we ease the upcoming notation a little bit. Let a type \((X, I)\) be labeled by \(j\) and define the enrollment share for income level \(I\):

\[E(I) = \int_X \mathbb{1}_{V_E \geq V_H} h(X|I) dX,\]

where \(\mathbb{1}_{V_E \geq V_H}\) is an indicator function capturing the education choice for each type \(j = (X, I)\).

Next, we define the completion rate by

\[C(I) = \frac{\int_X \mathbb{1}_{V_E \geq V_H} P(X, I, G(I)) h(X|I) dX}{E(I)},\]

which captures the share of enrolled students of parental income level \(I\) that actually graduate. We assume that these shares, as well as the probability of dropping out, \(P(X, I, G(I))\), are differentiable in the level of financial aid.

### 2.3 Government Problem and Optimal Policies

We now characterize the optimal financial aid schedule \(G(I)\). We denote by \(F(I)\) the unconditional parental income distribution, by \(K(X, I)\) the joint c.d.f. and by \(H(X|I)\) the conditional one; the densities are \(f(I), k(X, I),\) and \(h(X|I)\), respectively. The government assigns Pareto weights \(\tilde{k}(X, I) = \tilde{f}(I) \tilde{h}(X|I)\), which are normalized to integrate up to one.

Importantly, we assume that the government takes the tax-transfer system \(T(\cdot)\) as given and consider the optimal budget-neutral reform of \(G(I)\). Whereas the tax-transfer system is not changed if financial aid is reformed, a change in the financial aid schedule changes the size
and the composition of the set of individuals that go to college. This implies a change in tax revenue and transfer spending that directly feeds back into the available resource for financial aid.\footnote{We consider this as the more policy-relevant exercise than considering the joint optimal choice of $T(\cdot)$ and $G(I)$. Nevertheless, to complete the picture, in Section 7.3, we consider the joint optimal design of financial aid $G(I)$ and the tax-transfer system $T(\cdot)$. Further, we also explore jointly optimal merit and need-based financial aid in Appendix A.3 and Section 7.2.}

Taking the tax-transfer system as given, the problem of the government is

\[
\max_{G(I)} \int_{R^+} \int_X \max\{V^E(X, I), V^H(X, I)\} \tilde{k}(X, I) dXdI
\]

subject to the net present value government budget constraint:

\[
\int_{R^+} \int_X \mathcal{N}T_{NPV}^H(X, I) \mathbb{1}_{V_j^E < V_j^H} k(X, I) dXdI
\]

\[
+ \int_{R^+} \int_X \mathcal{N}T_{NPV}^G(X, I) \mathbb{1}_{V_j^E \geq V_j^H} P(X, I, G(I)) k(X, I) dXdI
\]

\[
+ \int_{R^+} \int_X \mathcal{N}T_{NPV}^D(X, I) \mathbb{1}_{V_j^E \geq V_j^H} (1 - P(X, I, G(I))) k(X, I) dXdI \geq \bar{F},
\]

where $\mathbb{1}_{V_j^E < V_j^H}$ and $\mathbb{1}_{V_j^E \geq V_j^H}$ are indicator functions capturing the education choice for each type $j = (X, I)$. The term $\bar{F}$ captures exogenous revenue requirements (e.g., spending on public goods) and exogenous revenue sources (e.g., tax revenue from older cohorts). Hence, $\bar{F} < 0$ could capture that the cohort for which we are reforming the financial aid schedule is effectively subsidized from other cohorts.

Now we consider a marginal increase in $G(I)$. As we show in Appendix A.1, it has the following impact on welfare:

\[
\frac{\partial E(I)}{\partial G(I)} \times \Delta T^E(I) + \frac{\partial C(I)}{\partial G(I)} \bigg|_{E(I)} \times E(I) \times \Delta T^C(I) - \tilde{E}(I) \left(1 - W^E(I)\right).
\]

The first two terms of (3) capture behavioral effects (i.e., changes in welfare that are due to individuals changing their behavior). The third term captures the mechanical welfare effect (i.e., the welfare effect that would occur for fixed behavior). We start with the latter.

The mechanical effect captures the direct welfare impact of the grant increase to infra-marginal students. The more students are inframarginal in their decision to go to college and the more of them who do not drop out, the higher are the immediate costs of the grant increase. The term $\tilde{E}(I)$ is the total discounted years of college attendance of income group $I$ and is defined as...
\[
\tilde{E}(I) = \int_X \mathbb{1}_{\text{V}_{E_j} > \text{V}_{E_j}} \left( \sum_{t=1}^4 \left( \frac{1}{1 + r} \right)^{t-1} \prod_{s=1}^t P_s(X, I, \mathcal{G}(I)) \right) h(X|I)dX.
\]

This captures the direct marginal fiscal costs of the grant increase.

Since the utility of these students is valued by the government, the costs have to be scaled down by a social marginal welfare weight (Saez and Stantcheva, 2016). We denote average social marginal welfare weight of inframarginal students with parental income \( I \) by \( W^E(I) \). Formally it is given by

\[
W^E(I) = \int_X \mathbb{1}_{\text{V}_{E_j} > \text{V}_{E_j}} \left( \sum_{t=1}^4 \beta^{t-1} U_c^E(c_t, l^E_t; X, I) \left( 1 + \frac{\partial r(i)}{\partial \mathcal{G}(I)} \right) \right) \prod_{s=1}^t P_s(X, I, \mathcal{G}(I)) \tilde{h}(X|I)dX / \rho_{f(I)} \tilde{E}(I),
\]

where \( \rho \) is the marginal value of public funds and \( U_c^E \) is the marginal utility of consumption.\(^{16}\) Thus, \( W^E(I) \) is a money-metric (appropriately weighted) average marginal social welfare weight. One difference from the standard concept applies here, however. One has to correct for the implied reduction in parental transfers that accompanies an increase in resources for college students. For each marginal dollar of additional grants, students only have an increase in consumption that is given by \( \left( 1 + \frac{\partial r(i)}{\partial \mathcal{G}(I)} \right) \). The share \( \frac{\partial r(i)}{\partial \mathcal{G}(I)} \) of the grant increase is offset by a reduction in parental transfers and does not effectively reach the student. Ceteris paribus, the stronger the crowding out of transfers, the lower are these welfare weights since fewer of the additional grants effectively reach students.\(^{17}\)

We now turn to the behavioral welfare effects in the first line of (3). The first term captures the change in tax revenues due to an increase in enrollment and \( \frac{\partial E(I)}{\partial \mathcal{G}(I)} \) captures the additional enrollees. Since these individuals are marginal in their enrollment decision, this change in their decision has no first-order effect on their utility. Therefore, we only have to track the effect on welfare through the effect on public funds. The term \( \Delta T^E(I) \) captures the the average increase in the NPV of net tax revenues for these marginal enrollees. Formally, it is given by

\[
\Delta T^E(I) = \int_X \mathbb{1}_{H_j \rightarrow E_j} \Delta T^E(X, I) h(X|I)dX / \int_X \mathbb{1}_{H_j \rightarrow E_j} h(X|I)dX,
\]

where \( \mathbb{1}_{H_j \rightarrow E_j} \) takes the value one if an individual of type \( j \) is marginal in her college enrollment decision with respect to a small increase in financial aid. By definition we have \( \int_X \mathbb{1}_{H_j \rightarrow E_j} h(X|I)dX = \frac{\partial E(I)}{\partial \mathcal{G}(I)}. \) \( \Delta T^E(X, I) \) is the (expected) fiscal externality of an individual of type \( (X, I) \): \( \Delta T^E(X, I) = \mathcal{N}T^E_{\text{NPV}}(X, I) - \mathcal{N}T^H_{\text{NPV}}(X, I) \).

\(^{16}\)For notational convenience but without loss of generality, we assume that the marginal utility of consumption is independent of the preference shock \( \varepsilon_t \).

\(^{17}\)Note that we are not accounting for parents’ utilities here. Doing so would basically imply an increase in the social welfare weights as not only the children but also the altruistic parents are benefiting from the grants. The change in parental transfers would have no impact on parent’s utility due to the envelope theorem.
There is a second behavioral effect due to endogenous college dropout. This second term in (3) captures the increase in tax revenue due to an increase in the completion rate of the inframarginal enrollees. The term \( \frac{\partial C(I)}{\partial G(I)} \bigg|_{E(I)} \) is the partial derivative of completion w.r.t. financial aid, holding \( E(I) \) constant. Therefore, the term \( \frac{\partial C(I)}{\partial G(I)} \bigg|_{E(I)} \times E(I) \) captures the amount of inframarginal enrollees who did not graduate in the absence of the grant increase but graduate now. Again, the envelope theorem applies and the change in their behavior has no first-order effect on their utility. However, there is a welfare effect through the change in public funds. \( \Delta T^C(I) \) captures the implied change in net fiscal contributions through the increased completion rate:

\[
\Delta T^C(I) = \int_X \Delta T^C(X, I) \frac{\partial P(X, I, G(I))}{\partial G(I)} h(X \mid I) dX
\]

where \( \Delta T^C(X, I) = N^T_G^N PV(X, I) - N^T_D^N PV(X, I) \). Finally, note that formula (3) is independent of the adjustment in labor supply during college as a response to the grant increase. This is an implication of the envelope theorem.

Evaluating (3) at the optimum and rearranging yields the following (implicit) expression for the optimal level of financial aid for students with parental income \( I \).

**Proposition 1.** The optimal level of financial aid at income level \( I \) is determined by

\[
G(I) = \eta^E(I) \Delta T^E(I) + \eta^C(I) \Delta T^C(I) E(I),
\]

where \( \eta^E(I) = \frac{dE(I)}{dG(I)} \) and \( \eta^C(I) = \frac{dC(I)E(I)}{dG(I)} \) are the semi-elasticities of enrollment and completion, that is, they capture the percentage point increase in enrollment and completion in response to a 1% increase in the level of financial aid.

The formula for optimal financial aid (5) has a very intuitive interpretation. Optimal financial aid is increasing in the effectiveness of increasing college attendance measured by \( \eta^E(I) \); such behavioral responses have been estimated in the literature exploiting financial aid reforms; see the discussion in Section 4.3.18 This effect is weighted by the fiscal externality created (i.e., the increase in tax payments). Intuitively, the size of the fiscal externality depends on the returns to college for marginal students, another parameter that has been estimated in different contexts in prior work. Likewise, optimal financial aid is increasing in the effectiveness of increasing the completion rate of inframarginal students and the implied fiscal externality.

Third, optimal financial aid is decreasing in the share of inframarginal students, capturing the marginal cost of increasing financial aid. These marginal costs, however, are scaled by the

18This behavioral effect is a policy elasticity as discussed in Hendren (2016).
value placed on college students' welfare. As we anticipated in the introduction, this force that is captured by the denominator plays a key role in determining the shape of the optimal $G(I)$ schedule. Intuitively, since for higher income levels, more individuals are inframarginal in their decision (which we know for sure at least given current policies), the direct marginal costs of extending financial aid for higher income levels are much higher.

Finally note that the crowding out of parental transfers, which is implicit in the welfare weight, also plays a role. If crowding out is stronger for one parental income level $I$ than for income level $I'$, then this should ceteris paribus make financial aid relatively less generous for parental income level $I$ as compared to $I'$.

Formula (5) expresses the optimal policy as a function of reduced-form elasticities and provides intuition for the main trade-offs underlying the design of financial aid.\footnote{Sometimes such formulas are labeled as sufficient statistics formulas. See Kleven (2018) for a discussion on the terminology in the literature.} It is valid without taking a stand on the functioning of credit markets for students, the riskiness of education decisions, or the exact modeling of how parental transfers are influenced by parental income and how they respond to changes in financial aid. Those factors, of course, influence the values of the reduced-form elasticities. For example, a tightening of borrowing constraints should increase the sensitivity of enrollment especially for low-income students.

However, note that all terms in the optimal financial aid formula are endogenous with respect to policies. Even if we know the empirical values for current policies, this is not enough to calculate optimal policies. For this purpose, a fully specified model is necessary. In Section 3, we consider a stylized simple model to obtain explicit analytical results. Our main approach to study optimal policies, however, is to study a structurally estimated model in Sections 4-7.

Besides studying the optimal level, our approach allows us to answer a related but different question: to what extent could small reforms of the current (e.g., US) financial aid system be self-financing through higher future tax revenue? We consider this as an interesting complementary question for at least three reasons. First, it may be easier to implement small reforms to the existing current federal financial aid system. Second, it points out whether there are potential Pareto-improving free-lunch policy reforms on the table which are independent of the underlying welfare function. Third, it directly informs policy makers about the effective fiscal costs of such policies.

**Corollary 1.** The fiscal return on one marginal dollar invested in financial aid for students with parental income $I$ is given by

$$R(I) = \frac{\frac{\partial E(I)}{\partial G(I)} \times \Delta T^E(I) + \frac{\partial C(I)}{\partial G(I)} \bigg|_{E(I)}}{E(I)} \times \Delta T^C(I) - 1.$$  

12
Proof. This directly follows from (3). Dividing by the mechanical effect and setting $W^E(I) = 0$.

This expression can be interpreted as the rate of return on one dollar invested in additional college subsidies at income level $I$. If it takes the value 0.2, it says that the government gets $1.20 in additional tax revenue for one marginal dollar invested into college subsidies. If it is -0.5, it implies that the government gets 50 cents back for each dollar invested: increasing subsidies by one dollar costs the government only 50 cents per dollar spent. Therefore, another way of interpreting (6) is to say that the effective cost of providing one more dollar to students of parental income level $I$ is equal to $-R(I)$ dollar.

3 Is Optimal Financial Aid Progressive? A Simple Model

Whereas the formula for the optimal financial aid in Proposition 1 is intuitive, it is not easy to infer under which conditions it implies that optimal financial aid is decreasing in parental income. This is contingent on how the different elasticities depend on parental income given the optimal schedule of financial aid.

To shed further light on the question of the desirability of progressivity from a theoretical perspective, the goal of this section is to derive conditions in a simplified framework. As a first step, we consider how Proposition 1 simplifies in the absence of a college dropout decision.

Corollary 2. In the absence of a college dropout decision, the optimal level of financial aid at income level $I$ is determined by

$$G(I) = \frac{\eta^E(I)\Delta T^E(I)}{\bar{E}(I)(1 - W^E(I))}. \tag{7}$$

Expression (7) is significantly simpler than (5). Besides the welfare weight, it only depends on (i) the share of inframarginal students $\bar{E}(I)$, (ii) the share of marginal students as captured by the semi-elasticity $\eta^E(I)$, and (iii) the fiscal externality for marginal students $\Delta T^E(I)$.

As stated in the previous section, empirically, it is well known that $\bar{E}'(I) > 0$. Enrollment is strongly associated with parental income even though current policies are need-based. This association in itself is ceteris paribus a force for need-based financial aid simply because the real fiscal costs of increasing financial aid for low-parental-income children are lower. Further, the share of inframarginal students should be associated with the semi-elasticity $\eta^E(I)$: the more individuals are already going to college the less individuals are “left over” as potential marginal students. Finally, the selection mechanism into college should tell us something

\[\text{Hence, even if we had knowledge about how these reduced-form elasticities vary with parental income for current policies, this would not allow us to infer how these values change if policies change.}\]

\[\text{The welfare weight and } \bar{E}(I) \text{ are defined in the same way, with the difference that } P_t(X, I, G(I)) \text{ equals one.}\]
about the returns to college for marginal students and therefore $\Delta T^E(I)$. We now turn to a simplified version of the model to further explore this.

**Simplified Environment.** We assume that preferences are linear in consumption and that labor incomes are taxed linearly at rate $\tau$. We consider a static problem. If individuals do not go to college, they earn income $y_h$. If they go to college, they pay tuition $F$ and earn $y_h(1+\theta)$. Individuals are heterogeneous in ability/returns to college $\theta$. There is no uncertainty. Further, individuals are heterogeneous in parental income $I$. If individuals go to college, they receive a parental transfer $tr(I)$ with $tr'(I) > 0$ and financial aid $G(I)$.

**Individual Problem.** If an individual decides against college, utility is given by $U^H = (1-\tau)y_h$. If an individual does go to college, utility is given by $U^C(\theta, I) = (1-\tau)y_h(1+\theta) - (F - G(I) - tr(I))$. For each income level $I$, we can define the ability of the marginal college graduate $\hat{\theta}(I)$, implicitly given by $U^H = U^C(\hat{\theta}(I), I)$. All types $(\theta, I)$ with $\theta \geq (<)\hat{\theta}(I)$ (do not) attend college.

Note that higher parental income here simply has the role of lowering the costs of college. This implies that high-parental-income children are more likely to select into college. This channel is reinforced if there is a positive association between $I$ and $\theta$.

**Government Problem and Optimal Financial Aid for a Given $I$.** To focus on the efficiency aspect of financial aid, we disregard distributional concerns and assume that the tax rate $\tau$ is exogenously given. In particular, we focus on a government that extracts as many resources as it can from its citizens and as a policy tool can use financial aid $G(I)$. This shuts down any redistributive case for progressive financial aid.\(^{22}\) The government’s problem reads as

$$
\max_{G(I)} \int_{\mathbb{R}_+} \int_{\theta} \hat{\theta}(I) \tau y_h \ dK(\theta|I)dF(I) + \int_{\mathbb{R}_+} \int_{\hat{\theta}(I)} \tau y_h (1+\theta) - G(I) \ dK(\theta|I)dF(I). \quad (8)
$$

Differentiating this objective with respect to $G(I)$ gives a rather tractable expression for optimal financial aid $G(I)$, which we state in the following proposition.

**Proposition 2.** The optimal financial aid schedule associated with (8) is given by

$$
G(I) = \tau (F - tr(I)) - y_h(1-\tau)^2 \frac{1 - K(\hat{\theta}(I)|I)}{k(\hat{\theta}(I)|I)}, \quad (9)
$$

\(^{22}\)We could work with a more general welfare function here and obtain the same results about progressivity. The key implied aspect of the revenue maximization is that the government does not have a desire to redistribute from children with high parental income to children with low parental income.
where $\tilde{\theta}(I) = \frac{\mathcal{F} - tr(I) - G(I)}{(1-\tau)y_H}$.

Proof. See Appendix A.4.

To understand the first term, note that $tr(I)$ acts like a parental price subsidy. Hence, $\mathcal{F} - tr(I)$ captures the costs of college for young individuals in the absence of government subsidies. A level of the subsidy that is $G(I) = \tau(\mathcal{F} - tr(I))$ would then imply that the share of the costs that are borne by the government equals the share of returns that are reaped by the government, which would resemble the classic “siamese twins” result in a setting with a continuous education choice as in Bovenberg and Jacobs (2005).

In this binary education choice model, such a level of the subsidy is a knife-edge case. It would be optimal whenever the highest ability level $\tilde{\theta}$ is equal to $\frac{\mathcal{F} - tr(I)}{y_H}$. In that case, from a first-best sense, college is only efficient for the highest ability level, since for this ability level the costs of education $\mathcal{F} - tr(I)$ are equal to the returns of education $\tilde{\theta}y_H$. If the government sets the level of financial such that $G(I) = \tau(\mathcal{F} - tr(I))$, then only these individuals with the highest ability go to college.

If, however, college is first-best efficient up to an interior ability level $\theta^*$ (i.e., $\tilde{\theta} > \theta^* = \frac{\mathcal{F} - tr(I)}{y_h}$), then the subsidy will no longer be as generous such that all individuals with $\theta > \theta^*$ go to college. Instead, we will have $\tilde{\theta}(I) > \theta^*$, that is, not all individuals that should go to college from a first-best perspective will go to college. The reason is that the government that solves (8) cannot target subsidies to marginal students only and at the margin has to pay more to the inframarginal students when increasing financial aid. How profitable an increase in grants is for the government also depends on the ratio of inframarginal students (to whom the government only pays more) over marginal students (from whom the government receives more than it pays, formally $\tau\tilde{\theta}(I) > G(I)$).\(^{23}\)

As a consequence, the optimal level of financial aid for a given income level $I$ depends on the shape of the hazard rate of the skill distribution. Such distributional properties are then also key for the question of how optimal financial aid varies with parental income.

Is Optimal Financial Aid Decreasing in Parental Income? We now proceed in two steps and first state a result on the progressivity if parental income and child’s ability are independently distributed.

**Corollary 3.** Assume that ability $\theta$ and parental income $I$ are independent, that is, $K(\theta|I) = K(\theta) \forall \theta, I$. Then the optimal financial aid schedule is progressive (i.e., $G'(I) < 0 \forall I$) if the distribution $K(\theta)$ is log concave.

Proof. See Appendix A.5.

\(^{23}\)Another way to put this is that education does not follow a first best rule because the government can partly tax the rents of inframarginal students.
Intuitively, the first term in (9) is decreasing in $I$. Recall that this term captures the ‘siamese twins logic’ of Bovenberg and Jacobs (2005). The higher parental income, the lower are the costs of college $F - tr(I)$ and hence, for a given rate of subsidization $\tau$, the lower is the overall level of the subsidy.

Since $\hat{\theta}'(I) < 0$, the second term is decreasing in $I$ if the inverse of the hazard rate of $K(\theta)$ is decreasing. As Bagnoli and Bergstrom (2005) point out, log-concavity of a density function is sufficient for an increasing hazard rate.\(^{25}\)

Hence, in the illustrative case in which parental income and child’s ability are iid, we have an important benchmark, where the selection mechanism through parental income in itself calls for progressive financial aid. Next we turn to the empirically more appealing case in which parental income and ability are positively associated.\(^{26}\)

**Corollary 4.** Assume that ability $\theta$ and parental income $I$ are positively associated in the sense that for $I' > I$, the distribution $K(\theta|I')$ dominates $K(\theta|I)$ in the hazard rate order, that is,

$$\forall \theta, I, I' \text{ with } I' > I : \frac{k(\theta|I)}{1 - K(\theta|I)} \geq \frac{k(\theta|I')}{1 - K(\theta|I')}.$$\(^{(10)}\)

Then the optimal financial aid schedule is progressive (i.e. $G'(I) < 0 \forall I$) if the conditional skill distributions $K(\theta|I)$ are log concave.

**Proof.** See Appendix A.6. \(\square\)

Note that condition (10) is stronger than first-order stochastic dominance (FOSD) but does imply that the skill distribution of higher parental income levels first-order stochastically dominates the skill distribution of lower parental income levels. FOSD of the skill distribution, however, does not automatically imply (10).\(^{27}\)

For the empirically plausible Pareto distribution, FOSD does imply dominance in the hazard rate order. Consider, for example, the specification $k(\theta|I) = \alpha(I) \frac{\theta}{\theta + \alpha(I)}$, where $\alpha(I)$ is the thickness parameter. Here we have $\frac{1 - K(\theta|I)}{k(\theta|I)} = \frac{\theta}{\alpha(I)}$ and hence if $\alpha'(I) < 0$, then the tail of the skill distribution of high-parental-income children is thicker and the FOSD property is fulfilled. Therefore, (10) is fulfilled.

\(^{24}\)Note that for this we need $tr'(I) + G'(I) > 0$, i.e. that financial aid is not too progressive. As our proof in Appendix A.5 shows, this is the case.

\(^{25}\)Log-concavity of a probability distribution is a frequent condition used in many mechanism design or contract theory applications, as this is "just enough special structure to yield a workable theory" (Bagnoli and Bergstrom, 2005).

\(^{26}\)As Carneiro and Heckman (2003, p.27) write: "Family income and child ability are positively correlated, so one would expect higher returns to schooling for children of high-income families for this reason alone." In a famous paper, Altonji and Dunn (1996) find higher returns to schooling for children with more-educated parents than for children with less-educated parents.

\(^{27}\)See, e.g., Shaked and Shanthikumar (2007, p.18).
The goal of this section was to show that under some rather weak assumptions, optimal financial aid is indeed decreasing in income. Whereas the simple model provides an interesting and intuitive benchmark, a richer empirical model is needed to give more concrete policy implications. In the next section we set up such a model and quantify it for the United States.

4 Quantitative Model and Estimation

We present our fully specified structural model in Section 4.1 and the estimation of the model and the data we use for that in Section 4.2. In Section 4.3 we show that the quantitative model performs very well in replicating patterns in the data and some non-targeted quasi-experimental evidence.

4.1 Quantitative Model

In this subsection, we present a fully specified version of the model presented in Section 2. We first specify the underlying heterogeneity in Section 4.1.1. We then present functional form assumptions and further modeling assumptions that concern the problem of the children once education is finished (Section 4.1.2) and the education decisions (Section 4.1.3). Finally, we provide a microfoundation for parental transfers by modeling the parental decision in Section 4.1.4.

4.1.1 Heterogeneity

We first specify the underlying heterogeneity. Besides parental income $I$, individuals differ in $X = (\theta, s, \text{ParEdu}, \text{Region})$, which captures ability, gender, their parents’ education levels, and the region in which they live.

4.1.2 Workforce Problem

Workers’ flow utility in the labor force is parameterized as

$$U^W(c_t, \ell_t) = \left(\frac{c_t - \ell_t^{1+\varepsilon_s}}{1+\varepsilon_s}\right)^{1-\gamma},$$

where the labor supply elasticity $\frac{1}{\varepsilon_s}$ is allowed to vary by gender. Individuals work until 65 and start at age 18 in case they decide to not enroll in college. Each year, individuals in the workforce make a labor-supply decision and a savings decision.28

Life-cycle wage paths depend on ability $\theta$, gender $s$, education $e$, and on a permanent skill shock that individuals draw upon finishing education and entering the labor market. To avoid

\[17\]
repeating ourselves and as this is a rather standard model element, we present the details of the parameterization in the wage estimation section Section 4.2.3.

A high-school graduate who chooses to forgo college immediately learns her permanent skill shock and enters the workforce.\footnote{We assume a no-borrowing constraint for high school graduates but our results are robust allowing for borrowing over the life-cycle.} We assume that these individuals receive transfers from their parents in the first period after labor market entry so that the initial level of assets $a_1 = tr^H$ is equal to the parental transfer. Let the value of entering the workforce directly out of high school be given by

$$V_1^H (X, a_1 = tr^H) = \mathbb{E} [V_1^W (X, e = H, a_1 = tr^H, w_1)] ,$$

where the expectation is taken over the permanent skill shock.

### 4.1.3 College Problem

We now consider decisions of individuals that are enrolled in college. We assume that students can choose to work part-time, full-time, or not at all. Formally, $\ell^E_t \in \{0, PT, FT\}$.

For flow utility in college we assume the following functional form:

$$U^E (c_t, \ell^E_t, X, \epsilon^E_t) = c_t^{1-\gamma} + \kappa_X + \zeta^E_t + \epsilon^E_t ,$$

where $\kappa_X + \zeta^E_t + \epsilon^E_t$ captures the nonpecuniary aspects of college. To follow the terminology of the literature (Cunha et al., 2005; Heckman et al., 2006) we mainly refer to $-(\kappa_X + \zeta^E_t + \epsilon^E_t)$ as psychic costs in the remainder of the paper. Concretely, $\kappa_X$ is the (potentially negative) amenity value of attending college. Workers of higher ability may find college easier and more enjoyable and therefore may have additional nonpecuniary returns to college. Furthermore, children with parents who attended college may find college easier, as they can learn from their parents’ experiences. Finally, we allow the amenity value of college to vary by an agent’s gender, to reflect differences in college-going rates across genders. We parameterize it as

$$\kappa_X = \kappa_0 + \kappa_\theta \log (\theta) + \kappa_{\text{fem}} I(s = \text{female}) + \kappa_{\text{ParEd} \text{ParEdu}} .$$

The term $-\zeta^E_t$ is the cost of working in college,\footnote{We normalize $\zeta^0 = 0$ w.l.o.g.} and $\epsilon^E_t$ is a shock associated with continuing college and working $\ell^E$ hours. This represents any idiosyncratic factors associated with staying in college and working that are not captured elsewhere in the model. We assume that the idiosyncratic preference shock, $\epsilon^E_t$, is distributed as a nested logit, with a separate nest for the three options involving continuing in college and a separate nest for dropping out of college. We denote the nesting parameter by $\lambda$ and the scale parameter by $\sigma^E$. 

$$\kappa_X = \kappa_0 + \kappa_\theta \log (\theta) + \kappa_{\text{fem}} I(s = \text{female}) + \kappa_{\text{ParEd} \text{ParEdu}} .$$
In contrast to the model in Section 2, we assume that graduation is stochastic if individuals decide to not drop out. Agents do not graduate and remain in college with probability \((1 - Pr^t_{Grad})\). This feature allows the model to accommodate heterogeneity in time until graduation without modeling the decision to accumulate college credits and graduate.

We can therefore write the choice-specific Bellman equation of an agent who is enrolled in college and works \(\ell^E_t \in \{0, PT, FT\}\) as

\[
V_t^E(\ell^E_t, X, I, a_t, \varepsilon^t_t) = \max_{c_t} \left\{ U^E(c_t, \ell^E_t, X, \varepsilon^E_t) + \beta \left\{ (1 - Pr^t_{Grad}) \mathbb{E} [V_{t+1}^E(X, I, a_{t+1}, \varepsilon_{t+1})] + Pr^t_{Grad} \mathbb{E} [V_{t+1}^W(X, e = G, a_{t+1}, w_{t+1})] \right\} \right\}
\]

subject to

\[
c_t = \ell^E_t \omega + a_t - a_{t+1} - \mathcal{F}_{Region} + \mathcal{G}(X, I)
\]

and

\[
a_{t+1} \geq \bar{a}_{t+1},
\]

where \(V_{t+1}^E(X, I, a_{t+1}, \varepsilon_{t+1})\) and \(\varepsilon_{t+1}\) are defined below. The term \(\mathbb{E} [V_{t+1}^W(X, e = G, a_{t+1}, w_{t+1})]\) is the expected value of being a college graduate in the workforce in year \(t + 1\), where the expectation is taken over the permanent skill shock.

We allow tuition, \(\mathcal{F}_{Region}\), to depend on the agent’s region. This allows the model to capture differences in tuition across geographic regions and is also helpful for identifying the parameters of the model. Children receive a lump sum parental transfer the first year they enroll in college, so initial assets are equal to parental transfers. Workers in college do not pay interest on their loans, to reflect the repayment schedule of the Stafford Loan program.

At the beginning of each period, the agent must either choose to drop out of college or choose any of the three labor supply and college combinations. Agents who drop out of college pay a nonpecuniary dropout cost \(\delta\) and then make consumption/saving and labor supply decisions for the remainder of their life. We can write an agent’s problem at the beginning of the period as:

\[
V_t^E(X, I, a_t, \varepsilon_t) =
\max \left\{ V_t^D(X, a_t) - \delta, V_t^{E,0}(X, I, a_t, \varepsilon^0), V_t^{E,PT}(X, I, a_t, \varepsilon^{PT}), V_t^{E,FT}(X, I, a_t, \varepsilon^{FT}) \right\},
\]

where \(\varepsilon_t\) is the vector of choice-specific preference shocks.

**Enrollment Decision.** Finally, we turn to the crucial enrollment decision. At the beginning of the model, children must decide whether to enter college or to enter the labor market directly. We assume that agents receive an idiosyncratic type I extreme value preference shock \(\eta\) with scale parameter \(\sigma^\eta\), which reflects idiosyncratic taste for college that is unreflected.
elsewhere in the model. Therefore, we can write an individual’s enrollment problem as a function of the parental transfer functions:

\[ V_1(X, I, tr^H, tr^E) = \max \left\{ V^H_1(X, a_1 = tr^H), \mathbb{E} \left[ V^E_1(X, I, a_1 = tr^E, \varepsilon_1) \right] + \eta \right\}. \]

We now turn to the parent’s problem.

4.1.4 Parent’s Problem

We model the parent’s life-cycle. Each year the parent makes a consumption/saving decision.\(^{31}\) The parent also chooses how much to transfer to the child dependent on the child’s education choice. Therefore, the parent has to trade off the utility of helping their child through parental transfers with their own consumption.\(^{32}\)

Parents make transfers to their child in the year in which a child graduates from high school. We assume that parents commit to a transfer schedule before the child’s initial enrollment shock, \(\eta\), and wage shock, \(v\), are realized. This simplifies the model solution considerably.\(^{33}\) For all years when the transfer is not given the parent simply chooses how much to consume and save.\(^{34}\) The parent’s Bellman equation and details on the calibration of life-cycle parental earnings are given in Appendix B.5. In the main body, we only elaborate on the element of the utility function that arises due to transfers.

In the year of the transfer, the parent receives utility from transfers. Let \(F(tr^H, tr^E, X, I)\) represent the expected utility the parent receives from the transfer schedule \(tr^H, tr^E\), conditional on a child with initial state space \((X, I)\). We assume that \(F\) consists of three components. First, parents are altruistic; they care about their child’s expected lifetime utility. Second, parents are paternalistic; they receive prestige utility if the child attends college. Finally, parents receive warm-glow utility from transfers that is independent of how the transfer affects the child’s utility or choices. Each of these three components helps us to match key features of the relationship between parental transfers, parental income, and the child’s problem. Allowing for utility from warm-glow helps us to match the gradient between parental income

\(^{31}\)For simplicity, we do not model the parent’s labor supply decision. The parent only chooses consumption, savings, and transfers.

\(^{32}\)Parent’s decisions only enter the child’s problem through parental transfers; we do not model fertility choice or parental investments in the child’s development. In Section 7.4 we consider an extension of the model in which a child’s ability is determined endogenously as a function of parental investment. We show that the optimal financial aid schedule is still highly progressive in this setting.

\(^{33}\)If not, the child will have to take into account how parental transfers will respond to their preferences and ability shocks which they partially reveal through their college choice.

\(^{34}\)The fact that parents provide all transfers based on the initial enrollment decision can give the incentive to strategically enroll for one year and then drop out directly only to obtain the larger parental transfer. This is one reason for why we incorporated the dropout costs \(\delta\), which makes such strategic behavior less attractive. As we show in Section 4.3, our model performs well regarding the dynamics of dropout and graduation.
and transfers.\textsuperscript{35} Allowing the parents to receive utility from altruism, which is probably the most standard element, allows for the possibility that changes in the financial aid schedule crowd out parental transfers. If financial aid increases, parental transfers may decrease in response. Allowing for paternalism allows us to match the level of college transfers relative to transfers for children who forgo college and adds an additional crowding-out element.

Specifically, we write the parent’s expected utility from transfers as

$$
F(tr^H, tr^E, X, I) = \omega E[V_1(X, I, tr^H, tr^E)] + E \left[ \xi_{ParEd} I_E + \phi \left( \frac{c_b + tr^e}{1 - \gamma} \right) \right]
$$

where $I_E$ is a dummy indicating that the child enrolls in college. The first term measures the parent’s utility derived from altruism. The term $\omega$ measures the weight the parent places on the child’s lifetime expected utility. The next term measures utility from paternalism. $\xi_{ParEd}$ captures the “prestige” value of having a child attending college. This captures any additional utility a parent gets from a child attending college in addition to the direct utility for the child. We allow this parameter to vary by the parent’s education level. The final term measures utility from warm-glow, where we adopt the the functional form commonly used in the literature (De Nardi, 2004). The parameter $\phi$ measures the strength of the warm-glow incentive, and $c_b$ measures the extent to which parental transfers are a luxury good.

### 4.2 Estimation and Data

To bring our model to the data, we make use of the National Longitudinal Survey of Youth 97 (henceforth NLSY97). A big advantage of this data set is that it contains information on parental income and the Armed Forces Qualification Test score (AFQT-score) for most individuals. The latter is a cognitive ability score for high school students that is conducted by the US army. The test score is a good signal of ability. Cunha et al. (2011), for example, show that it is the most precise signal of innate ability among comparable scores in other data sets. We use the NLSY97 for data on college-going, working in college, dropout, parental transfers, and grant receipts.\textsuperscript{36} Since individuals in the NLSY97 are born between 1980 and 1984, not enough information about their later-life earnings is available. We therefore also use the NLSY79 to better understand how earnings evolve throughout an agent’s life. Combining both data sets has proven to be a fruitful way in the literature to overcome the limitations

\textsuperscript{35}Even without warm-glow utility, parental transfers will generally be increasing in parental income as the marginal cost of transfers are higher for low income families. However, including warm-glow allows us to match the relationship between parental income and transfers much more closely. Given that the focus of this paper is the differences in college-going by parental income level, we chose to include warm-glow utility here in order to closely match this relationship.

\textsuperscript{36}We calculate parental transfers using the same method as Johnson (2013) which involves summing direct parental transfers and the monetary value of living at home if the individual lives with his parents.
of each individual data set; see Johnson (2013) and Abbott et al. (2018). The underlying assumption is that the relation between the AFQT score and wages has not changed over that time period. We use the method of Altonji et al. (2012) to make the AFQT scores comparable between the two samples and different age groups.

We define an individual as a college graduate if she has completed at least a bachelor’s degree. An individual is considered enrolled in college in a given year if they report being enrolled in college for at least six months in a given academic year. Individuals who report enrolling for at least one year in a four-year college but do not report a bachelor’s degree are considered dropouts. Agents who never enroll in college are considered as high school graduates. Since individuals in the NLSY97 turn 18 years old between 1998 and 2002, we express all US dollar amounts in year 2000 dollars. We drop individuals with missing values for key variables. We also drop individuals who take off one year or more of college before re-enrolling. These agents constitute 11% of the sample. For the variable Region, we consider the four regions for which we have information in the NLSY: Northeast, North Central, South, and West.

An overview of our calibration and estimation procedure is given in Table 1. First of all, to quantify the joint distribution of parental income and ability, we take the cross-sectional joint distribution in the NLSY97. We choose \( P_{t}^{\text{Grad}} \) as the fraction of continuing students in the NLSY97 who graduate each year. We assume that all agents in the model have to graduate after six years by setting \( P_{t}^{\text{Grad}} = 1 \). We then proceed in four steps:

1. We calibrate and preset a few parameters in Section 4.2.1.
2. We calibrate current US tax and college policies in Section 4.2.2.
3. We estimate the parameters of the wage function in Section 4.2.3.
4. Based on that, we estimate the parameters of the child’s and parent’s utility via maximum likelihood. The likelihood function is discussed in Section 4.2.4. In Section 4.2.5 we provide a discussion of identification and in Section 4.2.6, we present the likelihood estimates.

### 4.2.1 Calibrated Parameters

We set the risk-free interest rate to 3% (i.e., \( r = 0.03 \)) and assume that individuals’ discount factor is \( \beta = \frac{1}{1 + r} \). For the labor supply elasticity, we choose \( \varepsilon = 5 \) for men and \( \varepsilon = 1.66 \) for women, which imply compensated labor supply elasticities of 0.2 and 0.6, respectively.\(^{37}\)

Note that the value of the labor supply elasticity does not influence our results about optimal

\(^{37}\)See Blau and Kahn (2007) for a discussion of labor supply differences across gender. Our results are robust to assuming smaller gender differences in labor supply behavior and also larger differences. The labor supply elasticity is in general not a crucial parameter for optimal financial aid.
Table 1: Parameters and Targets

<table>
<thead>
<tr>
<th>Object</th>
<th>Description</th>
<th>Procedure/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(I)$</td>
<td>Marginal distribution of parental income</td>
<td>Directly taken from NSLY97</td>
</tr>
<tr>
<td>$(\theta, I)$</td>
<td>Joint and conditional distribution of innate abilities</td>
<td>Directly taken from NSLY97</td>
</tr>
<tr>
<td>$r = 0.03$</td>
<td>Interest Rate</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{Men} = 5$</td>
<td>Inverse Labor Supply Elasticity for Men</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{Women} = 1.66$</td>
<td>Inverse Labor Supply Elasticity for Women</td>
<td></td>
</tr>
<tr>
<td>$P_{t_1}^{Grad}$</td>
<td>Graduation Probabilities</td>
<td>Directly taken from NSLY97</td>
</tr>
<tr>
<td>$Wage$</td>
<td>Wage Parameters</td>
<td>Estimated from regressions</td>
</tr>
<tr>
<td>$L$</td>
<td>Stafford Loan Maximum</td>
<td>Value in year 2000</td>
</tr>
<tr>
<td>$T(y)$</td>
<td>Current Tax Function</td>
<td>Heathcote et al. (2017)</td>
</tr>
<tr>
<td>$G(\theta, I)$</td>
<td>Need- and Merit-Based Grants</td>
<td>Estimated from regressions</td>
</tr>
</tbody>
</table>

financial aid for given taxes in our benchmark because we calibrate wages from elasticities and income as in Saez (2001). We are more explicit about that in Section 4.2.3.

4.2.2 Calibration of Current Policies

To capture current tax policies, we use the approximation of Heathcote et al. (2017), which has been shown to work well in replicating the US tax code. Since this specification does not contain a lump-sum element, we slightly adjust this schedule (see Appendix B.1 for more details). For tuition costs, we take average values for the year 2000 from Snyder and Hoffman (2001) for the regions Northeast, North Central, South, and West, as they are defined in the NLSY. We also take into account the amount of money that is spent per student by public appropriations, which has to be taken into account for the fiscal externality. Both procedures are described in detail in Appendix B.2. The average values are $7,434 for annual tuition and $4,157 for annual public appropriations per student. Besides these implicit subsidies, students receive explicit subsidies in the form of grants and tuition waivers. We estimate how this grant receipt varies with parental income and ability in Appendix B.3 using information provided in the NLSY97. We find a strong negative effect of parental income on financial aid receipt. Additionally, we can capture merit-based grants by the conditional correlation of AFQT scores with grant receipt. Finally, we calibrate the exogenous budget element $\bar{F}$ in the following way. For the current U.S. polices, we calculate the present value of financial aid spending and the present value of tax revenues collected from the cohorts that we consider (born between 1980 and 1984 from the NLSY97) and obtain $\bar{F}$ from the difference between the two.

Finally, we make the assumption that individuals can only borrow through the public loan system. In the year 2000, the maximum amount for Stafford Loans per student was $23,000.
We examine the sensitivity of this assumption in Sections 7.1 and 7.6, where we show that (i) the optimal financial aid schedule is still highly progressive when borrowing constraints are relaxed and (ii) the policy implications are very similar if we reestimate the model with borrowing limits that depend on parental resources.

4.2.3 Wage Estimation

We specify and estimate wage life-cycle paths as follows. Our procedure first estimates labor earnings life-cycle profiles and then calibrates the respective wage profiles based on those estimates in a second step. Specifically, we use the following functional form for earnings $y$:

$$\forall e = H, G : \log y_{it}^e = \beta_{0e} + \beta_{1e} \log \theta_i + \beta_{2e} t + \beta_{3e} t^2 + \beta_{4e} t^3 + v_{ie}^e.$$ (12)

We estimate separate parameters for high school graduates and college graduates. The parameter $\beta_{0e}$ captures different returns to ability for agents of a given education level. The extent to which the college wage premium is increasing in ability is determined by the ratio $\frac{\beta_{2G}}{\beta_{2H}}$. We find a ratio larger than 1, which implies a complementary relationship between initial ability and education. Our estimates can be found in Appendix B.4. $v_{ie}^e$ is a random effect that captures persistent differences in wages conditional on the agent’s schooling choice. We assume that agents do not know the value of $v_{ie}^e$ at the beginning of the model, but that its value is revealed as soon as the agents finish their education and enter the labor market. Uncertainty over $v_{ie}^e$ creates uncertainty over an agent’s returns to college. After $v_{ie}^e$ there is no further uncertainty about an agent’s wage path.

The age earnings coefficients $\beta_{1e}, \beta_{2e}$ and $\beta_{3e}$ are education dependent but independent from gender. However, since we assume different labor supply elasticities for men and women, the implied wage life-cycle profiles will differ across gender because how a given earnings path maps into wages depends on the labor supply elasticity, see Appendix B.4 for details. The age coefficients are estimated from the NLSY79 since individuals from the NLSY97 are only observed until their mid-30s. Our approach to estimating the relationship between innate ability, education, and labor market outcomes is similar to Abbott et al. (2018) and Johnson (2013). Given that the approach is rather standard except that we estimate coefficients on earnings instead of wages to obtain more realistic income distributions, we only report the estimates in Appendix B.4. There, we also explain how we add Pareto tails to the implied income distributions to account for undersampling of top incomes in the NLSY and how we calibrate the wage process from the incomes process similarly as Saez (2001) in a static framework.

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38 Dropouts have the same wage parameters as high school graduates except for the constant term. This gives us a very good fit for the relative earnings of dropouts, consistent with the evidence in Lee et al. (2017).
In sum, this procedure pins down a stochastic distribution of potential life-cycle wage paths for each individual, which depend on gender, ability, and the education decisions. We demonstrate in Section 4.3 that we obtain life-cycle paths of earnings and wages which are consistent with the data.\footnote{We use these same parameter estimates to calculate life-cycle earnings for parents. We choose the idiosyncratic component of earnings, \( v_i^* \), to generate earnings at age 45 equal to the parental earnings levels we observe in the data.}

### 4.2.4 Likelihood Function

We estimate the remaining parameters with maximum likelihood. An agent’s likelihood contribution consists of 1) the contribution of their initial college choice, 2) the contribution of their labor supply and continuation decision each year in college, and 3) the contribution of their realized parental transfers. Let \( \Gamma \) denote a vector of structural parameters. The set of parameters estimated via maximum likelihood consists of the CRRA parameter, \( \gamma \), the set of parameters governing the amenity value of college and working in college, \( \kappa_X \) and \( \zeta \), the dropout cost, \( \delta \), the parameters governing the parent’s altruism, paternalism, and warm glow, \( \omega \), \( \xi_{ParEd} \), \( \phi \) and \( c_b \), the parameters governing the distribution of the college enrollment and working in college preference shocks: \( \sigma^\eta \), \( \sigma^E \) and \( \lambda \), and the standard deviation of the measurement error of parental transfers, \( \sigma_{tr} \).

We assume that parental transfers are measured with normally distributed measurement error. Assume that the econometrician observes transfers \( tr_{E,o}^* \), which differ from true transfers, \( tr_{E}^* \), by an error term \( e_{tr} \). Further, we assume this error term is normally distributed: \( e_{tr} \sim N(0, \sigma_{tr}) \). We suppress all dependencies for notational convenience. Then, given parameters \( \Gamma \), the likelihood contribution of an agent who graduates from college after \( T_{E}^i \) years, has a sequence of work in college decisions of \( \{ \ell_{E}^i \}_{t=1}^{T_{E}^i} \), and has observed college transfers \( tr_{E,o}^i \) is\footnote{The probability of this event in fact also depends on the graduation probabilities \( Pr_{Grad} \). But these are just constant factors in the likelihood, which is why refrain from putting them here.}

\[
\mathcal{L}_i \left( e_i = G, tr_{E,o}^i, \{ \ell_{E}^i \}_{t=1}^{T_{E}^i} | \Gamma \right) = \Pr (E) \phi \left( \frac{tr_{E}^* - tr_{E,o}^i}{\sigma_{tr}} \right) \frac{1}{\sigma_{tr}} \left( \prod_{t=1}^{T_{E}^i} \Pr (\ell_{E}^i) \right),
\]

where the probability of initially enrolling in college, \( \Pr (E) \), and the choice probability of not dropping out and working \( \ell_{E}^i \) in college, \( \Pr (\ell_{E}^i) \), are given by the extreme-value choice probabilities as

\[
\Pr (E) = \frac{\exp \left( V_1^E / \sigma^\eta \right)}{\exp \left( V_1^E / \sigma^\eta \right) + \exp \left( V_1^H / \sigma^\eta \right)}
\]
and

\[ Pr(\ell^E_t) = \frac{\exp\left(V_{t}^{E,\ell^E}/(\sigma^{\ell^E} \lambda)\right) \left(\sum_{l \in \{0, PT, FT\}} \exp\left(V_{t}^{E,l}/(\sigma^{\ell^E} \lambda)\right)\right)^{\lambda-1}}{\left(\exp(V^D - \delta/\sigma^{\ell^E})\right) + \left(\sum_{l \in \{0, PT, FT\}} \exp\left(V_{t}^{E,l}/(\sigma^{\ell^E} \lambda)\right)\right)^{\lambda}} \]

where \( \sigma^\eta \) and \( \sigma^{\ell^E} \) are parameters governing the variance of the enrollment shock and college working shock, respectively, and \( \lambda \) is a nesting parameter.

The likelihood contribution of an agent who drops out in year \( T^D_i \), has a sequence of work in college decisions of \( \{\ell^E_{it}\}_{t=1}^{T_{dropout}-1} \), and has observed college transfers \( tr^{E,o}_i \) is

\[
\mathcal{L}_i \left( e_i = D, tr^{E,o}_i, \{\ell^E_{it}\}_{t=1}^{T_{dropout}-1} | \Gamma \right) = Pr(E) \phi \left( \frac{tr^{E*}_i - tr^{E,o}_i}{\sigma^{str}} \right) \frac{1}{\sigma^{str}} \left( \prod_{t=1}^{T_{dropout}-1} Pr(\ell^E_{it}) \right) Pr(D_{T^D})
\]

where the probability of dropping out, \( Pr(D_{T^D}) \), is given by the extreme value choice probabilities as

\[
Pr(D_{T^D}) = \frac{\left(\exp\left(V^{D}_{T^D}/\sigma^{\ell^E}\right)\right)}{\left(\exp\left(V^{D}_{T^D}/\sigma^{\ell^E}\right)\right) + \left(\sum_{l \in \{0, PT, FT\}} \exp\left(V_{t}^{E,l}/(\sigma^{\ell^E} \lambda)\right)\right)^{\lambda}}.
\]

The likelihood function of an agent who enters the labor force directly and is observed with transfers \( tr^{H,o}_i \) is given by

\[
\mathcal{L}_i \left( e_i = H, tr^{H,o}_i | \Gamma \right) = (1 - Pr(E)) \phi \left( \frac{tr^{H*}_i - tr^{H,o}_i}{\sigma^{str}} \right) \frac{1}{\sigma^{str}}.
\]

We therefore choose the parameters \( \Gamma \) to maximize the log likelihood:

\[
\max_{\Gamma} \sum_{i} \log \mathcal{L}_i (\cdot | \Gamma).
\]

### 4.2.5 Identification

The parameter \( \gamma \) and the parameter governing the variance of the college-enrollment preference shock, \( \sigma^\eta \), play crucial roles in our analysis as they determine the extent to which increasing financial aid affects the college enrollment decision. Higher values of \( \sigma^\eta \) and lower values of \( \gamma \) imply a smaller elasticity of enrollment with respect to increases in financial aid. These parameters are jointly identified by the relationship between enrollment and parental income of otherwise similar individuals. Enrolling in college will generally imply lower net income while
enrolled in college and higher income later in life. To the extent that borrowing constraints are effective and that parental transfers are increasing in parental income, children from lower-income backgrounds will not be able to smooth consumption and therefore will have lower consumption in their early life. The parameter $\gamma$ determines the cost of not being able to smooth consumption early in life. A high value of $\gamma$ therefore implies low college enrollment for individuals close to the borrowing constraint.

Furthermore, exclusion restrictions in the grant function help us to identify the elasticity of college going with respect to financial aid. The number of siblings enters the formula for financial aid but does not affect the child’s utility in college or his earnings directly (Brown et al., 2012). Therefore, different college-going and working choices of similar agents with different number of siblings helps us to identify $\gamma$ and $\sigma^\eta$. Finally, tuition varies by region but region does not enter the earnings function or utility function. Therefore, similar to Heckman et al. (1998), variation in tuition levels creates variation in the value of college enrollment which helps us to identify $\gamma$ and $\sigma^\eta$.

Additionally, the extent to which poor students are more likely to work than rich students will be governed by $\gamma$; this tells us how much more students who are close to the borrowing constraint are willing to work relative to those who are not. As such, we can identify $\gamma$ by comparing the labor supply decisions of poor students with those from rich students.

The amenity value of college, $\kappa_X$, is identified by different rates of attending college by ability, gender, and parental education, after controlling for differences in utility coming from consumption and differences in future earnings. The parameters governing the value of working in college, $\zeta$, $\sigma^{\ell E}$, and $\lambda$, are jointly identified by variation in college labor supply choices across agents and across periods. Specifically, the parameter vector $\zeta$ is identified by different rates of working in college after controlling for differences in utility coming from consumption and differences in future earnings. The parameter governing the variance of the labor supply shock, $\sigma^{\ell E}$, is identified by variation in the timing of working in college decisions. For example, suppose that $\sigma^{\ell E} = 0$. Then the labor supply decisions of identical agents would be exactly the same in each period. A larger $\sigma^{\ell E}$ implies more variation in the labor supply choices of identical agents and across periods. The nesting parameter $\lambda$, is identified by the substitution patterns across labor supply decisions and dropping out.

The warm-glow parameters, $\phi$ and $c_b$, are identified by the relationship between parental income and parental transfers. A larger value of $\phi$ increases the derivative of parental transfers on parental income. Decreasing $c_b$ increases the level of transfers overall. Warm-glow utility only depends on the amount of transfers given, not on other things that may enter the child’s problem (i.e., ability, tuition, number of siblings). The degree to which parental transfers respond to different children’s characteristics will instead be determined by the strength of the altruism motive. Therefore, any differences in parental transfers across student characteristics
### Table 2: Maximum Likelihood estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>College Utility: $U^E(c, l) = \frac{c^{1-\gamma}}{1-\gamma} + \kappa_{\theta,d} + \zeta_{lE} + \varepsilon_{lt}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Curvature of Utility</td>
<td>$\gamma$</td>
<td>1.9</td>
</tr>
<tr>
<td>No work</td>
<td>$\zeta^0$</td>
<td>0</td>
</tr>
<tr>
<td>Part time</td>
<td>$\zeta^1$</td>
<td>-0.055</td>
</tr>
<tr>
<td>Full time</td>
<td>$\zeta^2$</td>
<td>-0.18</td>
</tr>
<tr>
<td>Standard Deviation of Enrollment Shock</td>
<td>$\sigma^E$</td>
<td>5.8*</td>
</tr>
<tr>
<td>Dropout cost</td>
<td>$\delta$</td>
<td>1.3</td>
</tr>
<tr>
<td>Standard Deviation of Working Shock</td>
<td>$\sigma^{lE}$</td>
<td>0.49*</td>
</tr>
<tr>
<td>Nesting Parameter</td>
<td>$\lambda$</td>
<td>0.59</td>
</tr>
<tr>
<td>Constant</td>
<td>$\kappa_0$</td>
<td>-0.44</td>
</tr>
<tr>
<td>Ability Interaction</td>
<td>$\kappa_{\theta}$</td>
<td>7.5*</td>
</tr>
<tr>
<td>Female Dummy</td>
<td>$\kappa_{\text{fem}}$</td>
<td>0.058*</td>
</tr>
<tr>
<td>Parental Education</td>
<td>$\kappa_{\text{ParEd}}$</td>
<td>1.1*</td>
</tr>
</tbody>
</table>

Parental Utility from Transfers:

$$F(tr^H, tr^E, \Omega_i) = \omega EV(\Omega_i | tr^H, tr^E) + \mathbb{E}\left[\xi_{\text{ParEd}}(\varepsilon_i = E) + \phi \frac{(c_b + tr^E)^{1-\gamma}}{1-\gamma}\right]$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altruism</td>
<td>$\omega$</td>
<td>3.5*</td>
</tr>
<tr>
<td>Prestige Constant</td>
<td>$\xi_0$</td>
<td>0.0101*</td>
</tr>
<tr>
<td>Parent’s Education Interaction</td>
<td>$\xi_{\text{ParEd}}$</td>
<td>0.0036*</td>
</tr>
<tr>
<td>Warm Glow Strength</td>
<td>$\phi$</td>
<td>0.14</td>
</tr>
<tr>
<td>Warm Glow Level</td>
<td>$c_b$</td>
<td>23.5</td>
</tr>
</tbody>
</table>

* we display 10,000 times the parameter value.

will identify $\omega$. For example, if students who face higher tuition levels generally receive higher parental transfers, this will identify $\omega$ and give us a sense of how much we expect parental transfers to be crowded out by financial aid. Parents’ paternalism parameters, $\xi_{\text{ParEd}}$, are identified by the ratio of college parental transfers to high school parental transfers. A higher value of $\xi_{\text{ParEd}}$ implies higher transfers for children going to college relative to transfers for children entering the labor force directly.

Finally, the parameter governing the standard deviation of observed parental transfers, $\sigma^{tr}$, is identified by the variance in observed parental transfers of identical agents.

### 4.2.6 Maximum Likelihood Estimates

The maximum likelihood estimates are shown in Table 2. In this section we discuss the estimates of several of the key parameters. This section is intentionally brief, as the magnitude of the parameters is difficult to interpret in a vacuum. In the following section, we compare the results from the model to the data and to reduced-form evidence of college-going as we believe this is a more informative way to assess the model’s performance.
The parameter $\gamma$ governs the curvature of the utility function with respect to consumption and plays a key role in determining an agent’s risk aversion. We estimate $\gamma = 1.93$, which is in the middle of the range of estimates from the literature. As mentioned earlier, $\gamma$, along with the variance of the college-going shock plays an important role in dictating the elasticity of college enrollment with respect to financial aid. In the next section, we show that this elasticity is consistent with estimates from the reduced-form literature.

The parameter vector $\zeta$ gives the amenity value of not working, working part-time, and working full-time in college. We normalize the amenity of not working in college to $\zeta_0 = 0$. Students pay a relative large nonpecuniary cost to work full-time in college and a pay a smaller nonpecuniary cost to work part-time. We estimate a nonpecuniary cost associated with dropping out of 1.3, implying that dropping out involves an additional utility cost in addition to potential forgone earnings. The parameters governing the amenity value of college are $\kappa_0, \kappa_{\theta}, \kappa_{\text{fem}},$ and $\kappa_{\text{ParEd}}$. Our estimates of these parameters imply that the nonpecuniary value of college is increasing in an agent’s ability and parental education. Furthermore, females receive a higher amenity value of college relative to men, reflecting the fact that women attend college in high numbers despite lower monetary returns than men.

The parameters in the bottom panel of Table 2 govern parental transfers. As we are the first paper to combine these transfer motives, it is difficult to directly compare our parameter estimates to those from the literature. However, all the parameters show the expected sign, indicating that parents are imperfectly altruistic, receive paternalistic utility when their children attend college and this effect is larger when parents have attended college themselves, and receive positive warm glow utility from giving parental transfers to their children. Finally, note that all estimates are statistically significant at the conventional levels.

4.3 Model Performance and Relation to Empirical Evidence

In order to assess the suitability of the model for policy analysis, we look at how well it replicates well-known findings from the empirical literature and especially quasi-experimental studies.

4.3.1 Model Fit

Enrollment, Graduation and Dropout. Figure 1 illustrates enrollment and graduation rates as a function of parental income and AFQT scores in percentiles, respectively. The solid lines indicate results from the model, and the dashed lines are from the data. The relationships in general are well fitted, though we slightly underestimate the parental income gradient on enrollment rates and the relationship between graduation and AFQT scores. The overall number of individuals who enroll in college is 40.0% in our sample and 39.5% in our model. In our model, 27.7% of agents graduate from college compared to 29.8% in the data.
Data from the US Census Bureau are very similar: in 2009 the share of individuals aged 25-29 holding a bachelor’s degree is 30.6% – a number that comes very close to our data, where we look at cohorts born between 1980 and 1984. Figure 18 in Appendix B.6 displays enrollment rates by gender. As we can see, the model matches differences in college enrollment rates by gender.

Figure 1: Graduation and Enrollment Rates

Notes: The solid (red) line shows simulated enrollment and graduation shares by parental income and AFQT percentile. This is compared to the dashed (black) line which shows the shares in the data.

To get a better sense of how dropout and graduation rates evolve over an individual’s time in college, Figure 2 shows graduation and dropout fractions over time in the model and the data. The solid red line and the dashed black line show the fraction of the total population that have graduated as a function of number of years of college completed in the model and the data, respectively. In both the model and the data, graduation rates are very low for students with less than three years of college. Graduation shares peak at four years before decreasing. The dashed-dotted blue line and the dotted green line show the fraction of students that drop out in each year in the model and data, respectively. Dropout shares are slightly downward sloping as a function of years in college in both the model and the data. This slope is slightly steeper in the model compared to the data.
Parental Transfers. Differences in parental transfers across parental income levels can also play a role in generating differential college-going rates across income groups. We analyze the fit of our model with respect to parental transfers in Figure 3. We can see that college transfers are strongly increasing in parental income in both the model and data, though our model slightly overestimates the relation in the data. The average college transfer for enrollees with below-median parental income is $47,000 in the model compared to $48,000 in the data, while the average college transfer for enrollees with above-median parental income is $60,000 in the model compared to $59,000 in the data. The model does a good job of matching the average level of high school transfers; the average high school transfers in the model and data are both roughly $39,800. While in our simulations high school transfers are increasing globally in parental income, parental transfers for high school graduates in the data are decreasing for the highest-income children.41

Working During College. We match average hours worked quite well. The average college student in our simulation works 16.23 hours per week compared to 17.39 in the data.42 We observe a weak negative relationship between parental income and working during college in the model and the data.

Earnings and College Premia. Table 3 analyzes the performance of the model with respect to earnings dynamics. We can only compare the model to the NLSY97 data up to

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41A reasonable suspicion is that this partly reflects measurement error because the set of high-income children who never enroll in college is relatively small. Our parameter estimates were robust ignoring this set of individuals in the estimation.

42Note that average hours of work are calculated using data from the entire year and thus include work during summer break.
Figure 3: College Transfers and Parental Income

Notes: Present value of parental transfers given by parents of college enrollees and non-enrollees in data (NLSY97) versus model.

<table>
<thead>
<tr>
<th>Age</th>
<th>High-School</th>
<th>College</th>
<th>Mean Earnings</th>
<th>College Premia</th>
<th>SD(log (y))</th>
</tr>
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Notes: Data based on NLSY97 with cohorts born between 1980 and 1984. Mean earnings expressed in year 2000 dollars. Most recent wave from 2015. Model based moment results represent results from estimated model. Zero and small earnings below $300 a month excluded. SD(log (y)) equal to standard deviation of log earnings. NLSY97 is top coded at income levels around $155,000.

Table 3: Earnings Dynamics

The simulated mean earnings across ages are very close to those in the data. As described in Section 4, we account for top-coding of earnings data by appending Pareto tails to the observed earnings distribution. As such, average earnings are slightly larger in model as compared to the data. We match college earnings premia very closely until around age 32. After that, the model and data diverge slightly as more and more college students reach top-coded earnings in the NLSY97.
In Figure 19 in Appendix B.7, we plot the implied earnings profiles in the model over the full range of ages.

The effect of the fatter right tails we include in the model can also be seen in the fit of standard deviation of log earnings. The simulated standard deviation of log earnings is 4-7 log points higher than that in the data from age 25 to age 34.

The college-earnings premium averaged across all ages greater than 25 in our model is 83%, that is, the average income of a college graduate is nearly twice as high as the average income of a high school graduate. This is well in line with empirical evidence in Oreopoulos and Petronijevic (2013); see also Lee et al. (2017). Doing the counterfactual experiment and asking how much the college enrollees would earn if they had not gone to college, we find a return of 12.5% for one year of schooling, which is in the upper half of the range of values found in Mincer equations (Card, 1999; Oreopoulos and Petronijevic, 2013).

4.3.2 Untargeted Moments

Responsiveness of Enrollment to Grant Increases. Many papers have analyzed the impact of increases in grants or decreases in tuition on college enrollment. Kane (2006) and Deming and Dynarski (2009) survey the literature. The estimated impact of a $1,000 increase in yearly grants (or a respective reduction in tuition) on enrollment ranges from 1 to 6 percentage points, depending on the policy reform and research design. A more recent study by Castleman and Long (2016) looks at the impact of grants targeted to low-income children. Applying a regression-discontinuity design for need-based financial aid in Florida (Florida Student Access Grant), they find that a $1,000 increase in yearly grants for children with parental income around $30,000 increases enrollment by 2.5 percentage points.

Simulating a $1,000 increase in financial aid for all individuals in our model leads to a 1.48 percentage point increase in overall enrollment rates and a 1.61 percentage point increase for students near the studied discontinuity in Castleman and Long (2016). Overall, our simulated elasticities are fairly consistent with these reduced-form estimates. This gives us confidence in our maximum likelihood estimates, especially given that these reduced form estimates were not targeted in estimation.

Importance of Parental Income. A well-known empirical fact is that individuals with higher parental income are more likely to receive a college degree (see also Figure 1). However, it is not obvious whether this is primarily driven by parental income itself or by variables correlated with parental income and college graduation. Using income tax data and a research design exploiting parental layoffs, Hilger (2016) finds that a $1,000 increase in parental income leads to an increase in college enrollment of 0.43 percentage points. To test our model, we increased parental income for each individual by $1,000 and obtained increases in college
enrollment by 0.24 percentage points. Our model predicts a moderate direct effect of parental income, smaller but in line with Hilger (2016).

Returns for Marginal Students. We find a return to one year of schooling of 12.1% for marginal students. This reflects that marginal students are of lower ability on average than inframarginal students and is also in line with Oreopoulo and Petronijevic (2013). A clean way to infer returns for marginal students is found in Zimmerman (2014). In his study, students are marginal with respect to academic ability, measured by a GPA admission cutoff. He finds that these students have earnings 22% higher than those just below the cutoff, when earnings are measured 8 to 14 years after high school graduation. We perform a similar simulation and make use of the fact that the NLSY also provides GPA data. In fact, our model gives a return to college of 24.1%, measured 8 to 14 years after high school graduation, for students with a GPA in this neighborhood.43

5 Results: Optimal Financial Aid

We now present our quantitative results for optimal financial aid. We first present the Utilitarian benchmark in Section 5.1. We next show that results are robust to the welfare function and also hold if the government only wants to maximize tax revenue in Section 5.2. We show that a larger degree of progressivity can be implemented in a Pareto-improving way in Section 5.3.

5.1 Optimal (Need-Based) Financial Aid

For our first policy experiment, we ask which levels of financial aid for different parental income levels maximize Utilitarian welfare. For this experiment, we do not change taxes or any other policy instrument but instead only vary the targeting of financial aid. Additionally, we work under the constraint that financial aid is nonnegative everywhere.44 At this stage, we leave the merit-based element of current financial aid policies unchanged, that is, we do not change the gradient of financial aid in merit and show the financial aid level for the median ability level. In Section 7.2, we show that our main result also extends to the case in which the merit-based elements are chosen optimally.

43 Finally, we do not account for differing rates of unemployment and disability insurance rates. Both numbers are typically found to be only half as large for college graduates (see Oreopoulo and Petronijevic (2013) for unemployment and Laun and Wallenius (2016) for disability insurance). Further, the fiscal costs of Medicare are likely to be much lower for individuals with a college degree. Lastly, we assume that all individuals work until 65 not taking into account that college graduates on average work longer (Laun and Wallenius, 2016). These facts would generally strengthen the case for an increase in college subsidies.

44 Relaxing this, one would get a negative subsidy at high parental income levels but nothing substantial changes in terms of results.
Figure 4: Optimal versus Current Financial Aid

Notes: Optimal financial aid with a Utilitarian welfare function and current financial aid in Panel (a). In Panel (b) we display the college graduation share by parental income group.

Figure 4(a) illustrates our main result for the benchmark case. Optimal financial aid is strongly decreasing in parental income. Compared to current policies, financial aid is higher for students with parental income below $80,000. This change in financial aid policies is mirrored in the change in college graduation, as shown in Figure 4(b). The total graduation rate increases by 2.8 percentage points to 32.4%. This number highlights the efficient character of this reform.

5.2 Tax-Revenue-Maximizing Financial Aid

One might be suspicious of whether the progressivity is driven by a desire for redistribution from rich to poor students that results in declining welfare weights.\textsuperscript{45} If this were the case, the question would naturally arise whether the financial aid system is the best means of doing so. However, we now show that the result holds even in the absence of redistributive purposes. We ask the following question: how should a government that is only interested in maximizing tax revenue (net of expenditures for financial aid) set financial aid policies? Figure 5(a) provides the answer: revenue-maximizing financial aid in this case is very progressive as well. Whereas the overall level of financial aid is naturally lower if the consumption utility of students is not valued, the declining pattern is basically unaffected. For lower parental income levels, revenue-maximizing aid is more generous than the current schedule, which implies that an

\textsuperscript{45}In fact, $1 - W^C(I)$, which is the relevant term for the formula, increases from around 0.32 to around 0.75 at the top, so by a factor of around 2.3. Note that this welfare weight is defined such that it accounts for crowding out of parental transfers. In fact, we find that crowding out is stronger for high parental income students. Going from the lowest to the highest parental income, the crowding out rate is monotonically increasing from 9% to 21%. The fact that $1 - W^C(I)$ increases by a factor of 2.3 is hence not only due to the Utilitarian welfare function but also due to the fact that an increase in financial aid reaches the poorest students to 91% while only to 79% for the kids with the highest parental income.
increase must be more than self-financing. We study this in more detail in Section 5.3. The implied graduation patterns are illustrated in Figure 5(b).

5.3 Pareto-Improving Reforms

As anticipated in Section 5.2, an increase in financial aid can be self-financing if properly targeted. The solid red line in Figure 6 illustrates the fiscal return as defined in (6), that is, the net effect on government revenue were financial aid for a particular income level to be increased by $1. Returns are positive for parental income between $0 and $23,000; the latter number corresponds to the 21st percentile of the parental income distribution. This result is striking: increasing subsidies for this group is a free lunch. An alternative would be to consider reforms where financial aid is increased for students below a certain parental income level. This case is illustrated by the dashed-dotted blue line in Figure 6. An increase in financial aid targeted to children with parental income below $44,000 – corresponding to the 47th percentile – is slightly above the margin of being self-financing.

6 Why Are Optimal Policies Progressive?

We have just shown in Section 5 that optimal financial aid is progressive, and more so than the current US policies. We have also shown that the results are not primarily driven by the desire to redistribute from richer to poorer students. In this section, we explore the key forces
determining the progressivity result. Recall that the change in welfare due to a small increase of $G(I)$ is given by (3)

$$\frac{\partial E(I)}{\partial G(I)} \times \Delta T^E(I) + \frac{\partial C(I)}{\partial G(I)} \bigg|_{E(I)} E(I) \times \Delta T^C(I) - \tilde{E}(I) (1 - W^E(I)).$$

We now illustrate the enrollment effect and the mechanical effect, evaluated at the current US system, in order to explain why welfare is increasing as the schedule becomes more progressive.

In this section we ignore the completion effect for brevity, as we only found a quantitatively very small contribution of it to financial aid policies.

Figure 7(a) plots the increase in enrollment for a $1,000 increase starting from the current financial aid system against parental income.

The curve is hump-shaped; slightly increasing at the beginning and then strongly decreasing. Middle-income children increase their enrollment rates most strongly. The decreasing elasticity contributes to the progressivity results at least after parental income levels of $40,000.

In contrast, the fiscal externality $\Delta T^E(I)$ is increasing in parental income because marginal enrollees from higher income households have higher returns.

We now turn to the mechanical effect. Figure 8 plots the share of inframarginal enrollees and graduates, which are the key determinants of the mechanical effect. As discussed above,
there is a strong parental income gradient, as the share of graduates (enrollees) increases from around 13% (19%) to around 51% (69%). Note that this basically implies that the marginal direct fiscal costs of a grant increase do increase by factor of 3-4 with parental income.

Both the increasing share of inframarginal students and the declining share of marginal students are important drivers of the progressivity result. The increasing share of inframarginal students is the more important driver: it varies by a factor of 3-4 whereas the share of marginal enrollees only varies by factor of 1.5 as we move from the lowest to the highest parental income. How robust are these results to changes in modeling assumptions? We first analyze the robustness about the shape of marginal students in Section 6.1. Here our focus is on the role of parental transfers, work during college and borrowing constraints. These elements should all have an impact on how responsive people are to financial aid. We then elaborate on which elements drive the increasing share of inframarginal students in Section 6.2. There, we provide a model-based decomposition to understand the increasing share of inframarginal students. We show that the correlation between parental income and ability and psychic costs are key drivers.

6.1 Enrollment Elasticity: Understanding the Shape

In this section, we analyze how the elasticity of college enrollment behaves under a number of alternative model specifications. First, in Figure 9(a) we recalculate the share of marginal students and tax revenue changes.

Notes: In (a), we plot the change in enrollment rates for a simulated $1,000 change in financial aid for each parental income level. The average (across all individuals in the sample) is 1.48 percentage points. In (b), we show the implied average fiscal externality across all students who are marginal w.r.t. the financial aid increase.
students when education-related borrowing constraints have been removed.\textsuperscript{47} This leads to a decrease in the share of marginal individuals across the income distribution, with the largest decrease for low-income students. This is intuitive: without borrowing constraints, individuals are less responsive to short-term financial incentives when choosing whether or not to enroll in college, and moreover, borrowing constraints are more important for low-income students. But even in a world with lax borrowing constraints, the shape of the marginal student function favors progressivity.

In Figure 9(b), we shut down working in college. The share of marginal students increases. This is intuitive, if college students cannot work during college, financial aid is more important for them. The shape of the marginal enrollee curve, however, is largely unaffected, which is in line with the fact that work during college is only weakly negatively correlated with parental income.

Figure 9(c) shows the share of marginal enrollees when parental transfers are exogenous and therefore are not crowded out by financial aid. As financial aid does not crowd out parental transfers in this scenario, increases in financial aid lead to larger increases in enrollment. The effect is increasing for most of the parental income distribution, reflecting that crowd out increases in parental income.

6.2 Relationship between Parental Income and College Enrollment

As we have seen, the positive relation between college graduation and parental income plays a crucial role in determining the optimal financial aid schedule. In this section, we perform a model-based decomposition exercise to better understand which factors drive this relationship.

\textsuperscript{47}Specifically, we allow individuals to borrow up to average tuition plus cost of room and board – $6,500 a year as in Johnson (2013).
between college graduation rates and parental income. For this decomposition, all changes to the model specification are cumulative. That is, each new model specification contains the same model alterations as the previous specification.

The simulated relationship between college graduation rates and parental income evaluated at the optimal level of grants is shown in the solid line in Figure 10. In this baseline case, college graduation rates are strongly increasing in parental income, even though the optimal financial aid schedule is highly progressive: 42% of students in the top quartile of parental income graduate from college compared to only 21% in the bottom quartile of parental income.

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48 Alternatively, we could have focused on enrollment instead of graduation. The implications are very similar.

49 Results are similar if the decomposition is performed for the current financial aid system.
One factor that leads to this positive relationship is the correlation between parental income and ability. Higher-ability students enjoy a larger college wage premium and have lower psychic costs of attending college. To understand the contribution of this correlation toward differential college-going rates by parental income, we simulate a version of the model in which we remove the correlation between parental income and ability by drawing each agent’s ability from the unconditional ability distribution. We can see from Figure 10 that the correlation between college graduation has reduced substantially, with 29% of children from the bottom quartile of the income distribution graduating from college compared to 38% of children from the top quartile of the income distribution.

Additionally, children with higher parental income tend to have lower psychic costs since parental education and parental income are positively correlated. We remove the relation between parental education and psychic costs in college by setting $\kappa_{ParEd} = 0$, in addition to removing the correlation between parental income and ability. After removing these differences in psychic costs, the relationship between parental income and college graduation has become slightly negative. The reason is that the progressive financial aid schedule now induces more children from low-income families to attend college relative to high-income families. These first two exercises show that the differences in psychic costs and ability across parental income groups play a large role in the parental income gradient in college graduation.

In our model, there are further factors which influence the parental income gradient in college education. The individual returns to college are not known at the time of the enrollment decision. As individuals are risk averse and as parents with higher income levels give higher transfers for students attending college, this riskiness of college is another mechanism which can generate a positive relationship between college and parental income. In addition to the modifications above, we remove this risk in the monetary return to college by simulating a version of the model in which each agent, with certainty, receives an ability draw of $\tilde{v}_{s,\theta}$, where $\tilde{v}_{s,\theta}$ is chosen to keep average conditional wage levels constant. Removing the riskiness of college leads to a slight increase in college graduation rates of students with low-income parents relative to those with high-income parents.

Next, we examine the role of borrowing constraints in generating this relationship by assuming that agents can borrow freely throughout their life cycle. This leads to a nearly 10 percentage point increase in the college graduation rate without a significant change in the correlation between parental income and college graduation. As the optimal grant schedule is highly progressive, removing the borrowing constraint does not lead to significant increases.

\footnote{Furthermore, we set $\kappa_0$ so that the average psychic cost of going to college is unchanged.}
in low-income college graduation rates relative to high-income students, even though students with high parental income receive higher parental transfers.\footnote{We also have performed the same decomposition with the current financial aid schedule. In this case removing borrowing constraints does significantly decrease the correlation between parental income and college graduation rates.}

Finally, we remove the relationship between parental transfers and parental income by simulating a version of the model in which all parents give transfers as if they earn the mean level of parental income. This has little effect on college going rates once these other elements that drive the parental income gradient have been shut down.

![Figure 10: Model-Based Decomposition of Parental Income Gradient](image)

Notes: We plot the share of college graduates given the optimal utilitarian financial aid for different model specifications. The solid red line represent the full model. For the dashed black line we simulate a model version for which we remove the correlation between ability and parental income. For the dashed-dotted blue line we simulate a model version for which we remove the correlation between the costs psychic and parental education on top. For the dotted pink line we simulate a model version for which on top removes any riskiness; i.e. education decisions are made under perfect foresight. For the dashed green line with circles we simulate a model version for which on top we remove all borrowing constraints. For the turquoise line with crosses we simulate a model version for which we equalize transfers across parental income levels conditional on education.

### 6.2.1 Decomposition: Recalculation of Optimal Policies

This decomposition has illustrated the key factors in the positive relation between parental income and college education. Removing the different elements from the model also affects the other forces that determine the optimal financial aid schedule. We therefore now simulate the respective optimal financial aid schedule. Figure 11 shows the implied optimal policies for each model specification.
First, the positive ability-income correlation does not drive the financial aid results (dashed black line). Although graduation rates flatten (see the black dashed line in Figure 10), there is an offsetting effect as low-income children now have higher ability and therefore larger returns, which increases fiscal externalities for low-income children. Policies are still progressive in this model.

Removing the correlation with psychic costs and eliminating income risk makes them – expectedly – less progressive, the effect is small, however. Next we eliminate borrowing constraints from the stripped-down model (with no interesting heterogeneity except for transfers; we come back to the issue of borrowing constraints in Section 7.1, where we shut them down without doing the previous steps.). This makes optimal policies less progressive but a significant gradient remains. Finally, the turquoise crossed line shows – finally – an almost zero slope, as transfers are equalized and we are in a world with no meaningful heterogeneity; thus, there is no longer a welfare argument for having need-based financial aid.\footnote{The turquoise crossed line in Figure 11 shows some noise and is not perfectly flat because there are still two (rather uninteresting) sources of heterogeneity left even in the last model: gender and region of origin. These two variables have a zero correlation with parental income but lead to the small bumps. Gender still enters psychic costs and region affects tuition. When we eliminate the two variables, we arrive at a flat schedule.}

### 7 Extensions

In this section we provide various extensions and robustness checks. First, we stick to the model in the main specification and elaborate on how policy implications for progressivity
would change if the government could (i) remove borrowing constraints in Section 7.1, (ii) optimally set the merit-based element in Section 7.2, and (iii) set the income tax schedule optimally.

In Section 7.4, we calibrate a version of the model where parents may respond to changes in the financial aid schedule by adjusting their investment in their child’s development. In Section 7.5, we calculate optimal financial aid when wages are determined in general equilibrium. Finally, in Section 7.6, we then discuss further issues such as income effects and parental incentives for earnings.

### 7.1 The Role of Borrowing Constraints

We have shown that optimal progressivity is not primarily driven by redistributive tastes but rather by efficiency considerations in Section 5.2. Given that our analysis assumes that students cannot borrow more than the Stafford Loan limit, the question arises whether these efficiency considerations are driven by borrowing limits that should be particularly binding for low-parental-income children. To elaborate on this question, we ask how normative prescriptions for financial aid policies change if students can suddenly borrow as much as they want (up to the natural borrowing limit, which is not binding). For this thought experiment, we first remove borrowing constraints and keep the current financial aid system. This will increase college enrollment and imply a windfall fiscal gain for the government. In a second step, we choose optimal financial aid but restrict the government to not use this windfall gain. As illustrated in Figure 12(a), optimal financial aid policies become a bit less progressive in this case. This is expected. More low-income children are close to the borrowing constraint in the baseline specification. When we remove borrowing constraints, redistributing funds towards these students becomes less attractive for the utilitarian social planner. Quantitatively, however, optimal policies are still very progressive even when borrowing constraints are removed.

We also reestimated a version of the model in which borrowing constraints varied by parental resources. We found that the optimal financial aid schedule was very similar to the baseline schedule. Details can be found in Appendix B.9. We also considered alternative unreported versions, where exogenous borrowing constraints depend differently on characteristics of the child and the parent. The policy implications were not affected much.

### 7.2 Merit-Based Financial Aid

In the benchmark in Section 5, we have assumed that the merit-based element of financial aid policies stays unaffected. We now allow the government to optimally choose the gradient in merit and parental income. Figure 13(a) shows that the need-based element is basically
Figure 12: Financial Aid and Graduation with Free Borrowing
Notes: The dashed-dotted (blue) line shows the optimal schedule when borrowing constraints have been removed. Optimal financial aid with borrowing constraints at their baseline levels and current financial aid are also shown for comparison in Panel (a). In Panel (b) we display the college graduation share by parental income group for each of the three scenarios.

unchanged. Figure 13(b) shows how optimal financial aid is increasing in AFQT score. Interestingly, the relation is only slightly upward sloping and very similar for different levels of parental income. The intuition for the rather flat relationship is simple: whereas higher ability levels have higher returns to college and therefore higher fiscal externality levels, they are also more likely to be inframarginal. These two forces roughly balance such that the optimal aid is almost flat in ability.

Figure 13: Optimal Need and Merit Based Financial Aid.
Notes: The dashed-dotted (blue) line shows the optimal schedule for the median ability level when we chose the merit-based element of financial aid optimally. Optimal financial aid with the merit-based component fixed at the baseline levels and current financial aid are also shown for comparison in Panel (a). In Panel (b) we display the optimal financial aid as a function of ability for various parental income levels.
7.3 Jointly Optimal Financial Aid and Income Taxation

The size of the fiscal externality of college education depends on the tax-transfer system in place. Our structural estimates took the current US tax-transfer system as given. An interesting question to ask is how optimal subsidies change when the tax-transfer system is chosen optimally. To address this, we enrich the optimal policy space such that the planner can also pick a nonlinear tax function $T(y)$, as is standard in the public finance literature (Piketty and Saez, 2013).\footnote{We abstract from education dependent taxation; for such cases please see Findelien and Sachs (2016) and Stantcheva (2017).}

First, the optimal formulas for the subsidy schedule are unchanged and still given by the formulas in Section 2. In Appendix A.7, we show what the endogenous extensive education margin implies for optimal marginal tax rate formulas.\footnote{The formula is therefore related to the formulas of Saez (2002) and Jacquet et al. (2013), where the extensive margin is due to labor market participation, or Lehmann et al. (2014) where the extensive margin captures migration.} For the sake of brevity, we discuss the theory only in the appendix and now move on to the quantitative implications of optimal taxes. We assume that agents are subject to borrowing constraints and the government only (besides the tax schedule) maximizes the need-based element of the financial aid schedule. Results are barely changed if borrowing constraints are relaxed, the merit-based element is chosen optimally as well, or both.

Figure 14(a) displays average tax rates in the optimal as well as the current U.S. system. Average tax rates are higher for most of the income distribution. Figure 14(b) illustrates optimal financial aid in the presence of the optimal tax schedule. First, notice that financial aid is significantly higher on average compared to the case with the current U.S. tax code. Higher income tax rates increase the fiscal externality, which increases the optimal level of financial aid. Second, the progressivity of optimal financial aid policies is preserved. Progressive taxation does not change the desirability of progressive financial aid policies. An unreported decomposition exercise shows that this is again driven by the increasing share of inframarginal students along the parental income distribution.

7.4 Endogenous Ability

Up to this point, we have assumed that a child’s ability at the beginning of the model, $\theta$, is exogenous. One might be concerned that parents may respond to changes in the financial aid schedule by adjusting their investment in their child’s development, therefore changing their child’s ability at the time of college entrance. To better understand how the optimal financial aid schedule would differ if ability were endogenous with respect to financial aid, we posit a
Notes: In Panel (a), the solid (red) line shows the optimal average tax rates when income tax and financial aid are both chosen optimally. The dashed (black) line shows the current schedule. In Panel (b) we display the optimal financial aid schedule when taxes are chosen optimally, the optimal financial aid when the tax function is held at its current schedule, and the current financial aid schedule.

model extension in which a child’s ability is determined endogenously as a function of parental investment.

Children are endowed with an initial ability at birth $\theta_0$. A child’s ability at the time of college, $\theta$, is produced as a function of the child’s initial ability and parental monetary investment, $\text{Invest}$. Specifically, we assume the following functional form, which is very similar to and based on the translog functional form employed in Agostinelli and Wiswall (2016)\textsuperscript{55}:

$$\theta = \ln A + \gamma_1 \ln \theta_0 + \gamma_2 \ln \text{Invest} + \gamma_3 \ln \theta_0 \ln \text{Invest} + \iota,$$

where $\iota$ is a normally distributed error that is unknown by the parent at the time of choosing $\text{Invest}$. We calibrate the parameters of the childhood ability production function to match the joint distribution of parental income and ability we observe in our data and selected moments from Agostinelli and Wiswall (2016). Details on the calibration are included in Appendix B.8.

Dahl and Lochner (2012) use changes in the EITC to instrument for family income and find that a $1000 increase in family income leads to an increase in ability scores by 6% of a standard deviation. We simulate an increase in yearly family income of parents by $1,000 in our model. The increase in income leads to an average increase in AFQT scores of 2.3% of a standard deviation across all children, and an increase of 5.4% of a standard deviation for

\textsuperscript{55}Agostinelli and Wiswall (2016) estimate a model of early childhood developments with multiple periods in which childhood skills are latent. Additionally, they use a broader concept of parental investment; the investment we refer to here is strictly monetary.
children in the lowest quintile. Therefore, the simulated responsiveness of ability with respect to parental income is slightly smaller than but in line with Dahl and Lochner (2012).

The optimal financial aid schedule, graduation rates, and ability levels with endogenous ability are shown in Figure 15. Panel 15(a) shows the new optimal financial aid schedule when ability is endogenous. Compared to the baseline case when ability is exogenous, the optimal aid schedule is now much higher, reflecting that increases in financial aid are now much more profitable for the government. With endogenous ability, increases in financial aid lead to increases in child ability, which increase tax payments of both marginal and infra-marginal children. The optimal aid schedule is still highly progressive. Panel 15(b) shows the graduation rates evaluated at the optimal aid schedule with endogenous ability. Switching to the optimal schedule leads to an increase in college graduation rates of over 10%, reflecting that 1) the optimal schedule is considerably more generous than the current schedule and 2) increases in financial aid lead to larger increases in college-going when ability is endogenous. Panel 15(c) shows the change in the relationship between parental income and ability as a result of switching from the current financial aid system to the optimal system with endogenous ability. Ability is measured in percentiles of AFQT scores where percentiles are evaluated at their current levels. We can see that switching to the optimal aid schedule leads to substantial increases in child ability, especially for children in the lower end of the parental income distribution.

One issue with the preceding analysis is that we have assumed that parents do not face borrowing constraints. Poor parents may be borrowing constrained while their children are young and therefore may not be able to increase investment in their children in response to changes in financial aid. To explore how borrowing constraints would affect the optimal policy, we assume that $P\%$ of parents without a college education cannot increase their investment in their children while the remainder of parents may choose their investment level without this constraint.\footnote{Caucutt and Lochner (2017) find that 20\% of parents with a high school degree and young children are borrowing constrained. Of course, borrowing constraints will also affect the investment decisions of parents who are not at the borrowing limit.} The optimal policy for a range of values of $P$ is displayed in Figure 16. We can see that the optimal progressivity of the system decreases as we increase the percentage of low-education families who are borrowing constrained. However, the optimal schedule remains more progressive than the current schedule in all cases.

7.5 General Equilibrium Effects on Wages

Our analysis abstracted from general equilibrium effects on relative wages. Accounting for these effects would imply that the effects of financial aid on enrollment might be mitigated in
the long run: if more individuals go to college, the college wage premium should be expected to decrease because of an increase in the supply of college educated labor (Katz and Murphy, 1992). This in turn would mitigate the initial enrollment increase. To investigate the role of general equilibrium effects on our results, we recalculate the optimal financial aid schedule under the assumption that wages are determined in equilibrium. We assume firms use a CES production function that combines total efficiency units of labor supplied by skilled and unskilled workers, implying that wages are determined by the ratio of skilled to unskilled labor. We assume an elasticity of substitution between skilled and unskilled workers of 2. Details on the approach can be found in Appendix B.1.0.

The optimal financial aid schedule and graduation rates with general equilibrium wages are shown in Figures 17(a) and 17(b). We can see that the overall amount of aid has decreased...
Figure 16: Financial Aid, Graduation and Ability Levels with Endogenous Ability and Parental Borrowing Constraints

Notes: In Panel (a), each line shows the optimal financial aid with endogenous ability when $P$ percent of low-education parents are borrowing constrained and therefore cannot adjust their child’s ability in response to changes in financial aid. In Panel (b) we display the college graduation share for each of these scenarios. Panel (c) shows the relationship between parental income and ability in each scenario. Ability is measured in percentiles of the AFQT distribution before financial aid is re-optimized.

slightly as the fiscal externality of college has been scaled down by general equilibrium wage effects. However, the optimal aid schedule with endogenous wages is just as progressive as in the case with exogenous wages. Thus, while general equilibrium wages dampen the effectiveness of financial aid overall, they do not lead to dramatic changes in the relative benefit of financial aid increases for students of different parental income levels. Hence, whereas the overall (average) generosity of the optimal financial aid schedule is slightly lower, the implications for how financial aid should vary with parental income are unchanged.\footnote{Our results are, hence, consistent with the important earlier paper(s) by Heckman et al. (1998). They find that GE effects dampen the effectiveness of tuition subsidies, and in our case the average level of financial aid is also affected.}
Figure 17: Financial Aid and Graduation with General Equilibrium Wages

Notes: The dashed-dotted (blue) line shows the optimal schedule when wages are determined in equilibrium. Production is CES between skilled and unskilled workers with an elasticity of substitution of 2. Optimal financial aid with exogenous wage rates and current financial aid are also shown for comparison in Panel (a). In Panel (b) we display the college graduation share by parental income group for each of the three scenarios.

7.6 Further Aspects

Income Effects We assumed away income effects on labor supply for simplicity. How would our analysis change if income effects were taken into account? If an increase in financial aid decreases borrowing (which should happen unless individuals are borrowing constrained), it lowers the stock of student debt when individuals enter the labor market. If leisure is a normal good, this implies lower earnings of college graduates. This then triggers a reduction in tax revenue and makes the increase in financial aid less desirable ceteris paribus.

A reasonable upper bound is to assume that a $1 increase in financial aid leads to a $1 decrease in borrowing for inframarginal students. This approximately decreases the stock of student debt by $1. How can we expect this to affect lifetime earnings? Imbens et al. (2001) use a survey of lottery players to estimate income effects and find that a $1 increase in wealth triggers a decrease in earnings of about $0.11. For a marginal tax rate of 30%, this would imply a loss in tax revenue of about $0.03. Thus, the marginal fiscal costs of increasing financial aid would be increased by 3% according to this simple back-of-the-envelope calculation. Whereas this generally is an effect that policy makers should bear in mind, we conjecture that it does not weaken our result about the optimal progressivity of financial aid. To weaken our progressivity result, the effect would have to be larger for lower parental income levels. However, since low-parental-income students are actually more likely to be borrowing constrained, the income effects should be smaller for them and we conjecture that the opposite is true and income effects would instead reinforce our results.
Parental Earnings Incentives An increase in the progressivity of financial aid can, of course, have adverse effects on parental incentives. Need-based financial aid implies an increase in effective marginal tax rates and can lower parental labor supply (or reported income more generally) which lowers tax revenue and increases financial aid payments. In an earlier version of this paper (Findeisen and Sachs, 2015), we elaborated this potential additional fiscal effect when considering the fiscal effects of financial aid reforms. The quantitative extent of these effects turned out to be rather modest.

Welfare Weights We have shown that the progressivity result is not driven by the Utilitarian objective in Section 5.2 where we considered a tax revenue maximizing planner that puts zero welfare weights on all students. But what if society puts higher welfare weights on children with higher parental income? Zoutman et al. (2016) ask the question for which social welfare weights the Dutch tax schedule is optimal and find that these welfare weights are increasing in income for low incomes. Applying our analysis to the Dutch financial aid system should take that into account and this would be a force against progressivity. Following a very similar approach for the US, Lockwood and Weinzierl (2016) have shown that welfare weights are declining in income. We therefore refrain from considering such an alternative welfare function for the US.

8 Conclusion

This paper has analyzed the normative question of how to optimally design financial aid policies for students. We find the very robust result that optimal financial aid policies are strongly progressive. This result holds for different social welfare functions, assumptions on credit markets for students, and assumptions on income taxation. Moreover, we find that a progressive expansion in financial aid policies could be self-financing through higher tax revenue, thus benefiting all taxpayers as well as low-income students directly. Financial aid policies are a rare case with no classic equity-efficiency trade-off because a cost-effective targeting of financial aid goes hand in hand with goals of social mobility and redistribution. We do think that our results can be used for policy recommendations according to the criteria of Diamond and Saez (2011):58 the economic mechanism is empirically relevant and of first order importance to the problem. The result is very robust. Finally, progressive financial aid systems are clearly implementable, as they are already in use in all OECD countries.

58 Diamond and Saez (2011) write in their abstract: "We argue that a result from basic research is relevant for policy only if (a) it is based on economic mechanisms that are empirically relevant and first order to the problem, (b) it is reasonably robust to changes in the modeling assumptions, (c) the policy prescription is implementable (i.e., is socially acceptable and is not too complex)."
A Theoretical Appendix

A.1 Derivation of Equation 3

The Lagrangian for the government’s problem reads as:

\[ \mathcal{L} = \int_{\mathbb{R}_+^2} \max\{V^E(X,I), V^H(X,I)\} \tilde{k}(X,I) dX dI + \rho \int_{\mathbb{R}_+^2} \int_{\mathbb{X}} N^T_{NPV}^H(X,I) \mathbb{1}_{V^E_j < V^H_j} k(X,I) dX dI \]
\[ + \int_{\mathbb{R}_+^2} \int_{\mathbb{X}} N^T_{NPV}^G(X,I) \mathbb{1}_{V^E_j \geq V^H_j} P(X,I, G(I)) k(X,I) dX dI \]
\[ + \int_{\mathbb{R}_+^2} \int_{\mathbb{X}} N^T_{NPV}^D(X,I) \mathbb{1}_{V^E_j \geq V^H_j} ((1 - P(X,I, G(I))) k(X,I) dX dI - \bar{F}) \].

The derivative w.r.t. \( G(I) \) is given by:

\[ \frac{\partial \mathcal{L}}{\partial G(I)} = \int_{\mathbb{X}} \mathbb{1}_{V^E_j \geq V^H_j} \frac{\partial V^E(X,I)}{\partial G(I)} \tilde{h}(X|I) dX \]
\[ + \rho \int_{\mathbb{X}} \left\{ P(X,I, G(I)) \frac{\partial N^T_{NPV}^G(X,I)}{\partial G(I)} + (1 - P(X,I, G(I))) \frac{\partial N^T_{NPV}^D(X,I)}{\partial G(I)} \right\} h(X|I) dX \]
\[ + \rho \int_{\mathbb{X}} \mathbb{1}_{H_j \rightarrow E_j} \left\{ P(X,I, G(I)) N^T_{NPV}^G(X,I) + (1 - P(X,I, G(I))) N^T_{NPV}^D(X,I) \right\} h(X|I) dX \]
\[ - N^T_{NPV}^H(X,I) \mathbb{1}_{V^E_j \geq V^H_j} \tilde{h}(X|I) dX \]
\[ + \rho \int_{\mathbb{X}} \frac{\partial P(X,I, G(I))}{\partial G(I)} \left( N^T_{NPV}^G(X,I) - N^T_{NPV}^D(X,I) \right) \tilde{h}(X|I) dX \]

Recall that \( \mathbb{1}_{H_j \rightarrow E_j} \) takes the value one if an individual of type \( j \) is pushed over the college enrollment margin due to a small increase in financial aid.

The first term captures the direct utility increase of inframarginal enrollees due to receiving more financial aid. The second term captures the direct fiscal effect of paying more financial aid to inframarginal students. The third term captures the fiscal effect of additional enrollees. The fourth effect captures the fiscal effect due to the increase in the completion rate of inframarginal students. The implied change in the enrollment and dropout rate has no direct first-order effect on welfare: individuals that are marginal in their decision to enroll or not and to continue studying or drop out, were just indifferent between the two respective options, hence this change in behavior has no effect on their utility.

The definitions of \( E(I) \) and \( \Delta T^E(I) \) directly imply that the third term equals the enrollment effect in (3) multiplied by \( \rho \). The definitions of \( \Delta T^C(I) \), \( E(I) \) and \( C(I) \) directly imply that the fourth term equals the completion effect in (3) multiplied by \( \rho \).
Now it remains to be shown that the first and second term are equal to the mechanical effect in (3). The application of the envelope theorem implies that the first term reads as

\[
\int_{\chi} \mathbb{1}_{V_j \geq V_j^H} \sum_{t=1}^{4} \beta^{t-1} \prod_{s=1}^{t} P_t(X, I, G(I)) U_c^E(c_t, I_t^E; X, I) \left(1 + \frac{\partial tr(X, G(I))}{\partial G(I)}\right) \tilde{h}(X|I) dX. \tag{17}
\]

The last term, using the definitions of $\mathcal{N}T^G_{NPV}(X, I)$ and $\mathcal{N}T^D_{NPV}(X, I)$, can be written as

\[
-\rho \int_{\chi} \sum_{t=1}^{4} \frac{1}{1 + r} \prod_{s=1}^{t} P_t(X, I, G(I)) \tilde{h}(X|I) dX. \tag{18}
\]

Adding (17) and (18), using the definition of the social marginal welfare weight yields equation 3.

A.2 More General Version of Equation 3 with Annual Dropout Decisions

We now show the generalization in which individuals can drop out each period. For this case, we have to distinguish between individuals that drop out in different periods. Hence, for the education decision we have: $e \in \{H, G, D_1, D_2, D_3, D_4\}$, where $D_t$ implies that individuals drop out at the beginning of year $t$. Accordingly we can define the net fiscal contribution of an individual of type $(X, I)$ that drops out in period $t$ by $\mathcal{N}T^E_t(X, I)$:

\[
\mathcal{N}T^E_t(X, I) = P_t(X, I, G(I)) \mathcal{N}T^G_{NPV}(X, I) + (1 - P_t(X, I, G(I))) \mathcal{N}T^D_t(X, I)
\]

and for $t = 2, 3$:

\[
\mathcal{N}T^E_t(X, I) = P_t(X, I, G(I)) \mathcal{N}T^E_{t+1}(X, I) + (1 - P_t(X, I, G(I))) \mathcal{N}T^D_t(X, I).
\]
The Lagrangian for the government’s problem reads as:

\[ \mathcal{L} = \int_{\mathbb{R}^+} \int_{\chi} \max \{ V^E(X, I), V^H(X, I) \} \tilde{k}(X, I) dX dI \]

\[ + \rho \left\{ \int_{\mathbb{R}^+} \int_{\chi} N^T^H_{NPV}(X, I) 1_{V^E_j < V^H_j} k(X, I) dX dI \right. \]

\[ + \int_{\mathbb{R}^+} \int_{\chi} N^T^G_{NPV}(X, I) 1_{V^E_j \geq V^H_j} \frac{4}{\prod_{t=1}^{4}} P_t(X, I, \mathcal{G}(I)) k(X, I) dX dI \]

\[ + \int_{\mathbb{R}^+} \int_{\chi} N^T^{D1}_{NPV}(X, I) 1_{V^E_j \geq V^H_j} (1 - P_1(X, I, \mathcal{G}(I))) k(X, I) dX dI \]

\[ + \int_{\mathbb{R}^+} \int_{\chi} N^T^{D2}_{NPV}(X, I) 1_{V^E_j \geq V^H_j} P_1(X, I, \mathcal{G}(I)) (1 - P_2(X, I, \mathcal{G}(I))) k(X, I) dX dI \]

\[ + \int_{\mathbb{R}^+} \int_{\chi} N^T^{D3}_{NPV}(X, I) 1_{V^E_j \geq V^H_j} P_1(X, I, \mathcal{G}(I)) (1 - P_3(X, I, \mathcal{G}(I))) k(X, I) dX dI \]

\[ + \int_{\mathbb{R}^+} \int_{\chi} N^T^{D4}_{NPV}(X, I) 1_{V^E_j \geq V^H_j} P_1(X, I, \mathcal{G}(I)) (1 - P_4(X, I, \mathcal{G}(I))) k(X, I) dX dI - \vec{F} \}. \]

The FOC for \( \mathcal{G}(I) \) shares the same basic structure as (16). However, here the fiscal effects due to change in dropout behavior are more involved:\(^{50}\)

\[ \rho \int_{\chi} \prod_{t=1}^{3} P_t(X, I, \mathcal{G}(I)) \frac{\partial P_t(X, I, \mathcal{G}(I))}{\partial \mathcal{G}(I)} (N^T^G_{NPV}(X, I) - N^T^{D4}_{NPV}(X, I)) h(X|I) dX \]

\[ + \rho \int_{\chi} \prod_{t=1}^{2} P_t(X, I, \mathcal{G}(I)) \frac{\partial P_t(X, I, \mathcal{G}(I))}{\partial \mathcal{G}(I)} (N^T^{D4}_{NPV}(X, I) - N^T^{D3}_{NPV}(X, I)) h(X|I) dX \]

\[ + \rho \int_{\chi} P_1(X, I, \mathcal{G}(I)) \frac{\partial P_1(X, I, \mathcal{G}(I))}{\partial \mathcal{G}(I)} (N^T^{D3}_{NPV}(X, I) - N^T^{D2}_{NPV}(X, I)) h(X|I) dX \]

\[ + \rho \int_{\chi} \frac{\partial P_1(X, I, \mathcal{G}(I))}{\partial \mathcal{G}(I)} (N^T^{D2}_{NPV}(X, I) - N^T^{D1}_{NPV}(X, I)) h(X|I) dX, \]

In short term notation, similar to that in (3), we can write

\[ \sum_{t=1}^{4} \frac{\partial \text{Con}_t(I)}{\partial \mathcal{G}(I)} \bigg|_{E_t(I)} \Delta T^\text{Con}_t(I) E_t(I) \]

where \( \text{Con}_t(I) \) is the share of those enrollees with parental income \( I \) in period \( t \), that continue studying to year \( t + 1 \). It is defined by

\[ \text{Con}_t(I) = \frac{E_{t+1}(I)}{E_t(I)} \]

\(^{50}\)Again, changes in dropout behaviour have no direct welfare effect due to the envelope theorem.
for $t = 1, 2, 3$ and
\[
Con_4(I) = \int_X \mathbb{1}_{V_j \geq V_H} \prod_{s=1}^4 P_s(X, I, G(I)) h(X|I) dX / E_4(I),
\]
where
\[
E_1(I) = E(I) = \int_X \mathbb{1}_{V_j \geq V_H} h(X|I) dX.
\]
and
\[
E_t(I) = \int_X \mathbb{1}_{V_j \geq V_H} \prod_{s=1}^{t-1} P_s(X, I, G(I)) h(X|I) dX.
\]
Finally, the changes in tax revenue are defined by:
\[
\Delta T^{Con,t}(I) = \int_X \Delta T^{Con,t}(X, I) \frac{\partial P(X, I, G(I))}{\partial G(I)} h(X|I) dX
\]
where
\[
\Delta T^{Con,t}(X, I) = \mathcal{N}T_{NPV}^G(X, I) - \mathcal{N}T_{NPV}^D(X, I).
\]
Hence, the equivalent to equation 3 is given by:
\[
\frac{\partial E(I)}{\partial G(I)} \times \Delta T^E(I) + \sum_{t=1}^4 E_t(I) \frac{\partial Con_4(I)}{\partial G(I)} \bigg|_{E_t(I)} \Delta T^{Con,t}(I) - \tilde{E}(I) \left( 1 - W^E(I) \right).
\]

A.3 Merit-Based Policies

Our approach is more general and can be extended to condition financial aid policies on other observables like academic merit or jointly on the combination of parental income and academic merit. In fact, in our empirical application we will allow the government to also target financial aid policies on a signal of academic ability. Suppose the government can observe such a signal of academic ability like the SAT score. We take that factor out of the vector $X$ and label it $\theta$. For notational simplicity, we will still denote the vector without $\theta$ by $X$; in this case $X$ includes all factors influencing the college decision except for parental income and the measure of academic ability. Suppose we are interested in deriving the optimal policy schedule which conditions on need- and merit-based components jointly. Formally, the government maximizes over $G(I, \theta)$. The derivation of the optimal financial aid policy schedule is analogous to the derivation of $G(I)$ and yields:
\[
G(I, \theta) = \frac{\eta^E(I, \theta) \Delta T^E(I, \theta) + \eta^C(I, \theta) \Delta T^C(I, \theta) E(I, \theta)}{E(I, \theta) (C(I, \theta) - (1 - C(I, \theta)) t_d) (1 - W^E(I, \theta))}
\] (19)
where all terms are evaluated at a parental income-ability pair \((I, \theta)\).

How should we expect optimal financial aid to vary with academic ability, holding parental income fixed? At first glance, one may expect that the optimal grant \(G(I, \theta)\) is increasing in \(\theta\) as the returns to college education should increase in \(\theta\), which boosts the fiscal externality. By conditioning on ability directly, the government can implicitly guarantee that marginal students have a certain minimum expected return to college attendance, circumventing some of the potential problems of a pure need-based system. Working against this, is that among higher ability students there are (likely) more inframarginal students: i.e. they opt for college in any financial aid system. Our empirical model in Section X will shed light on this first question, which has no clear theoretical answer.

### A.4 Proof of Proposition 2

The first-order condition for \(G(I)\) is given by

\[
k(\hat{\theta}(I)|I) \frac{\partial \hat{\theta}(I)}{\partial G(I)} \left( \tau y_h \hat{\theta}(I) - G(I) \right) - \left( 1 - K(\hat{\theta}(I)|I) \right) = 0
\]

Hence

\[
G(I) = \frac{\left( 1 - K(\hat{\theta}(I)|I) \right)}{k(\hat{\theta}(I)|I) \frac{\partial \hat{\theta}(I)}{\partial G(I)}} + \tau y_h \hat{\theta}(I).
\]

Now use

\[
\frac{\partial \hat{\theta}(I)}{\partial G(I)} = -\frac{1}{y_H(1 - \tau)}
\]

Hence

\[
G(I) = \tau y_h \hat{\theta}(I) - y_H(1 - \tau) \frac{\left( 1 - K(\hat{\theta}(I)|I) \right)}{k(\hat{\theta}(I)|I)}
\]

Now substitute for the threshold:

\[
G(I) = \tau y_h \left( \frac{F - tr(I) - G(I)}{(1 - \tau)y_H} \right) - y_H(1 - \tau) \frac{\left( 1 - K(\hat{\theta}(I)|I) \right)}{k(\hat{\theta}(I)|I)}
\]

hence

\[
G(I) = \frac{\tau y_h \left( F - tr(I) \right) - y_H(1 - \tau) \frac{\left( 1 - K(\hat{\theta}(I)|I) \right)}{k(\hat{\theta}(I)|I)}}{1 + \frac{\tau}{1 - \tau}},
\]

which yields the result.
A.5 Proof of Corollary 3

Differentiating (9) w.r.t. $I$ yields:

$$G'(I) = -\tau tr'(I) + (1 - \tau) \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)} (tr'(I) + G'(I))$$

where we used $\tilde{\theta}(I) = \frac{F-tr(I)-G(I)}{(1-\tau)y_H}$, and therefore $\tilde{\theta}'(I) = \frac{-tr'(I)-G'(I)}{(1-\tau)y_H}$. Solving for $G'(I)$ we get

$$G'(I) = \frac{-\tau tr'(I) + (1 - \tau) \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)} (tr'(I))}{1 - (1 - \tau) \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)}}$$

which proves Corollary 3 since by assumption $tr'(I) > 0$ and log concavity of the skill distribution implies $\frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)} < 0$.

A.6 Proof of Corollary 4

Differentiating (9) w.r.t. $I$ yields:

$$G'(I) = -\tau tr'(I) + (1 - \tau) \frac{\partial \left( \frac{1-K(\tilde{\theta}(I)|I)}{k(\tilde{\theta}(I)|I)} \right)}{\partial \theta(I)} (tr'(I) + G'(I)) - y_H (1 - \tau)^2 \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial I} \bigg|_{\theta = \tilde{\theta}(I)}$$

Hence we obtain

$$G'(I) = \frac{-\tau tr'(I) + (1 - \tau) \frac{\partial \left( \frac{1-K(\tilde{\theta}(I)|I)}{k(\tilde{\theta}(I)|I)} \right)}{\partial \theta(I)} (tr'(I)) - y_H (1 - \tau)^2 \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial I} \bigg|_{\theta = \tilde{\theta}(I)}}{1 - (1 - \tau) \frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)}}$$

which proves Corollary 4 since by assumption $tr'(I) > 0$, log concavity of the skill distribution implies $\frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial \theta(I)} < 0$ and we assumed

$$\frac{\partial \left( \frac{1-K(\theta(I))}{k(\theta(I))} \right)}{\partial I} > 0 \forall \theta, I.$$
We restrict the tax function to be only a function of income and to be independent of the education decision. This tax problem can either be tackled with a variational or tax perturbation approach (Saez, 2001; Golosov et al., 2014; Jacquet and Lehmann, 2016) or with a restricted mechanism design approach for nonlinear history-independent income taxes that we explore in Findeisen and Sachs (2017).

We here provide a heuristic version of the former approach within our model. For notational convenience, we consider the model of Section 2 with the assumption that individuals can only dropout at the beginning of period 3.

Consider an increase of the marginal tax by an infinitesimal amount \( dT' \) in an income interval of infinitesimal length \([y^*, y^* + dy]\). As a consequence of this reform, all individuals with \( y > y^* \) face an increase of the absolute tax level of \( dT' dy \). The tax reform therefore induces a mechanical increase in welfare of

\[
\Delta W_{MR}(y^*) = \rho dT' dy \sum_{t=1}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} h_{t,H}(y) dy \times s_H
\]

\[
+ \rho dT' dy \sum_{t=3}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} h_{t,D}(y) dy \times s_D
\]

\[
+ \rho dT' dy \sum_{t=5}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} h_{t,G}(y) dy \times s_G
\]

through the tax revenue increase (in net present value). \( h_{t,e}(y) \) is the density of income of individuals with education level \( e \) in period \( t \) and \( s_e \) is the overall share of individuals with education level \( e \). Both, the income densities and the education shares are endogeneous w.r.t. to taxes, we get to this below.

Note that this increase in tax payment also has mechanical effects on individual utilities which adds up to the following welfare effect

\[
\Delta W_{MU}(y^*) = dT' dy \sum_{t=1}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} E(U_c|y_t = y) h_{t,H}(y) dy \times s_H
\]

\[
+ dT' dy \sum_{t=3}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} E(U_c|y_t = y) h_{t,D}(y) dy \times s_D
\]

\[
+ dT' dy \sum_{t=5}^{T} \left( \frac{1}{1 + r} \right)^{t-1} \int_{y^*}^{\infty} E(U_c|y_t = y) h_{t,G}(y) dy \times s_G.
\]

Now we turn to the endogeneity of education shares. First of all some individuals will change their initial enrollment decision. We define \( 1_{H_{j \rightarrow E_j}} \) to take the value one if an individual of type \( j \) is marginal in the enrollment decision w.r.t. to a one dollar tax increase for earnings.
above $y^*$. Then, the welfare effect of individuals changing their enrollment decision due to a small increase in $T'(y^*)$ is given by:

$$\Delta W_E(y^*) = \rho dT' \int_{R_{+}} \int_{\chi} 1_{H_{i} \rightarrow E_{j}} \left\{ N^E_{NPV}(X, I) - N^H_{NPV}(X, I) \right\} h(X | I) dXdI.$$

Similarly, the probability to continue college and not drop out is endogenous w.r.t. taxes, i.e. we have $P(X, I, G(I), T(\cdot))$. The change in welfare due to the change in dropout behavior, with some abuse of notation, is simply given by:

$$\Delta W_D(y^*) = \rho \int_{y^*}^{\infty} \int_{y^*}^{\chi} \int dy \left\{ N^G_{NPV}(X, I) - N^D_{NPV}(X, I) \right\} h(X | I) dXdI.$$

Finally, an increase in the marginal tax rate also affects labor supply behavior for individuals within the interval $[y^*, y^* + dy]$. Individuals within this infinitesimal interval change their labor supply by

$$\frac{\partial y^*_t}{\partial T'} dT' = -\varepsilon_{y^*, 1 - T'} \frac{y^*_t}{1 - T'} dT'.$$

Whereas this change in labor supply has no first-order effect on welfare via individual utilities by the envelope theorem, it has an effect on tax revenue, which is given by:

$$\Delta W_L(y^*) = \frac{T'(y^*)}{1 - T'(y^*)} y^* \varepsilon_{y^*, 1 - T'} \times dT' \times$$

$$\left( s_H \sum_{t=1}^{T} \left( \frac{1}{1 + r} \right)^{t-1} h_{t, H}(y^*) + s_D \sum_{t=3}^{T} \left( \frac{1}{1 + r} \right)^{t-1} h_{t, D}(y^*) + s_G \sum_{t=5}^{T} \left( \frac{1}{1 + r} \right)^{t-1} h_{t, G}(y^*) \right).$$

Since this reform must not have any non-zero effect on welfare if the tax system is optimal, we have to have

$$\Delta W_{MR}(y^*) + \Delta W_{MU}(y^*) + \Delta W_E(y^*) + \Delta W_D(y^*) + \Delta W_L(y^*) = 0 \quad (20)$$

which provides an implicit characterization of $T'(y^*)$.

Finally, the optimal level for the lump-sum element of the tax schedule $T(0)$ is implicitly characterized by

$$\Delta W_{MR}(0) + \Delta W_{MU}(0) + \Delta W_E(0) + \Delta W_D(0) = 0.$$
This optimal tax approach is related to the formulas of Saez (2002) and Jacquet et al. (2013), where the extensive margin is due to labor market participation, or Lehmann et al. (2014) where the extensive margin captures migration.\footnote{Further papers are Schener (2014) where the extensive margin captures the decision to become an entrepreneur and Kleven et al. (2009) who consider the extensive margin of secondary earner to study the optimal taxation of couples.}

To implement this formula numerically, we follow a guess and verify approach. Hence, we start with a guess for the tax schedule and then evaluate (20).\footnote{In fact a more complicated version of (20) which accounts for dropout behavior in every period and also accounts for stochastic graduation.} We then slightly adjust $T'(y^*)$ to make (20) closer to zero (but keep $\Delta W_{MR}(y^*) + \Delta W_{MU}(y^*) + \Delta W_E(y^*) + \Delta W_D(y^*)$ fixed, i.e. we only adjust $\Delta W_L(y^*)$). We then calculated the new allocation for this adjusted schedule and evaluate (20) for income levels again and so on. We proceed until convergence.\footnote{In each iteration, we also optimally choose the financial aid schedule $\mathcal{G}(I)$ given the tax schedule in the respective iteration.}

## B Estimation and Calibration

### B.1 Current Tax Policies

We set the lump sum element of the tax code $T(0)$ to minus $1,800 a year. For average incomes this fits the deduction in the US-tax code quite well.\footnote{Guner et al. (2014) report a standard deduction of $7,350 for couples that file jointly. For an average tax rate of 25\% this deduction could be interpreted as a lump sum transfer of slightly more than $1,800.} For low incomes this reflects that individuals might receive transfers such as food stamps.\footnote{The average amount of food stamps per eligible person was $72 per month in the year 2000. Assuming a two person household gives roughly $1,800 per year. Source: \url{http://www.fns.usda.gov/sites/default/files/pd/SNAPsummary.pdf}}

### B.2 Tuition Fees and Public Costs of Colleges

First, we categorize the following 4 regions:

- \textbullet\ Northeast: CT, ME, MA, NH, NJ, NY, PA, RI, VT
- \textbullet\ North Central: IL, IN, IA, KS, MI, MN, MO, NE, OH, ND, SD, WI
- \textbullet\ South: AL, AR, DE, DC, FL, GA, KY, LA, MD, MS, NC, OK, SC, TN, TX, VA, WV
- \textbullet\ West: AK, AZ, CA, CO, HI, ID, MT, NV, NM, OR, UT, WA, WY

We base the following calculations on numbers presented by Snyder and Hoffman (2001). Table 313 of this report contains average tuition fees for four-year public and private universities.
According to Table 173, 65% of all four-year college students went to public institutions, whereas 35% went to private institutions. For each state we can therefore calculate the average (weighted by the enrollment shares) tuition fee for a four-year college. We then use these numbers to calculate the average for each of the four regions, where we weigh the different states by their population size. We then arrive at numbers for yearly tuition & fees of $9,435 (North East), $7,646 (North Central), $6,414 (South) and $7,073 (West). For all individuals in the data with missing information about their state of residence, we chose a country wide population size weighted average of $7,434.

Tuition revenue of colleges typically only covers a certain share of their expenditure. Figures 18 and 19 in Snyder and Hoffman (2001) illustrate by which sources public and private colleges finance cover their costs. Unfortunately no distinction between two and four-year colleges is available. From Figures 18 and 19 we then infer how many dollars of public appropriations are spent for each dollar of tuition. Many of these public appropriations are also used to finance graduate students. It is unlikely that the marginal public appropriation for a bachelor student therefore equals the average public appropriation at a college given that costs for graduate students are higher. To solve this issue, we focus on institutions “that primarily focus on undergraduate education” as defined in Table 345. Lastly, to avoid double counting of grants and fee waivers, we exclude them from the calculation as we directly use the detailed individual data about financial aid receipt from the NLSY (see Section B.3). Based on these calculations we arrive at marginal public appropriations of $5,485 (Northeast), $4,514 (North Central), $3,558 (South), $3,604 (West) and $4,157 (No information about region).

B.3 Estimation of Grant Receipt

Grants and tuition subsidies are provided by a variety of different institutions. Pell grants, for example, are provided by the federal government. In addition, there exist various state and university programs. To make progress, similar to Johnson (2013) and others, we go on to estimate grant receipt directly from the data.

Next, we estimate the amount of grants conditional on receiving grants as a Tobit model:

$$ gr_i = \alpha^{gr} + f(I_i) + \beta_4^{gr} \text{AFQT}_i + \beta_5^{gr} \text{depkids}_i + \epsilon_i^{gr}. $$

(21)

where $f(I_i)$ is a spline function of parental income and $\epsilon_i^{gr}$ represents measurement error. Besides grant generosity being need-based (convexly decreasing), generosity is also merit-based as $\beta_4^{gr} > 0$ and increases with the number of other dependent children (besides the considered student) in the family.
Table 4: OLS for Grants

<table>
<thead>
<tr>
<th></th>
<th>AFQT</th>
<th>Dependent Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>39.40***</td>
<td>321.75**</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(5.03)</td>
<td>(106.39)</td>
</tr>
</tbody>
</table>

N=968. * p ≤ 0.10, ** p ≤ 0.05, *** p ≤ 0.01.

B.4 Wage Paths

Recall that the regression equation reads as

\[ \forall e = H, G : \log y_{it}^e = \beta_0^e + \beta_1^e \log \theta_i + \beta_2^e t + \beta_3^e t^2 + \beta_3^e t^3 + v_i^e. \]

We estimate the age coefficients \( \beta_{11}, \beta_{12}, \beta_{13} \) using panel data from the NLSY79 since individuals in the NSLY97 are too young (born between 1980 and 1984) such that we can infer how wages evolve once individuals are older than 35.

In the second step, we build the transformed variable \( \tilde{\log y}_{it}^e = \log y_{it}^e - \beta_1^e t - \beta_2^e t^2 - \beta_3^e t^3 \), which takes out age affects from yearly log incomes. Using the NLSY97, we estimate the relationship of log income with gender and log AFQT, estimating separate models and coefficients by education level. We use a random-effects estimator and assume normality, yielding education specific variances for \( v_i^e \). The estimates are displayed in Table 5. There is a significant college premium in the model, although the high-school constant is larger, because we have used education dependent age profiles.

Table 5: Regressions: Income

<table>
<thead>
<tr>
<th>College Educated</th>
<th>Female</th>
<th>Log AFQT</th>
<th>Education Constant</th>
<th>Variance ( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>-0.14***</td>
<td>0.47***</td>
<td>3.06***</td>
<td>0.42</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.35)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High-School Educated</th>
<th>Female</th>
<th>Log AFQT</th>
<th>Education Constant</th>
<th>Variance ( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>-0.25***</td>
<td>0.31***</td>
<td>7.11***</td>
<td>0.36</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.35)</td>
<td></td>
</tr>
</tbody>
</table>

Random effect models, estimated with NLSY9. Dependent variable is log yearly income, cleaned for age effects. Age effects are obtained by estimating a cubic polynomial on the NLSY79. These age coefficients are available upon request. N=10,165 (College) and N=19,955 (High-School). * p ≤ 0.10, ** p ≤ 0.05, *** p ≤ 0.01.

Next, we explain how to go from the estimated income to the wage profiles. The reason why we do not estimate wage profiles directly is that we append Pareto tails to the income dis-
tribution on which more reliable information is available. Top incomes are underrepresented in the NLSY as in most survey data sets. Following common practice in the optimal tax literature (Piketty and Saez, 2013), we therefore append Pareto tails to each income distribution, starting at incomes of $150,000. We set the shape parameter \( \alpha \) of the Pareto distribution to 1.5 for all income distributions.

Next we describe the mapping from \( y \) to \( w \) as in Saez (2001). Given the utility function we assume with no income effects, in each year individuals solve a static labor supply problem where optimal labor supply in that year only depends on the current wage (which evolves over the life-cycle) and marginal tax distortions. It is easy to show that the first-order condition for an individual facing a marginal tax rate schedule is

\[
\ln w = \frac{\varepsilon + \tau}{1 + \varepsilon} \ln y - \frac{1}{1 + \varepsilon} \ln(\lambda (1 - \tau)),
\]

if the tax function is of the form \( T(y) = y - \rho y^{1-\tau} \). Using the estimates from the regression model, we can express the wage for a given type (age, gender, ability, education) as at age \( t \):

\[
\ln w_{it} = \frac{\varepsilon + \tau}{1 + \varepsilon} \left( \beta_0^{\text{es}} + \beta_{\theta}^{\text{es}} \log \theta_i + \beta_{t}^{\text{es}} + \beta_{t2}^{\text{es}} + \beta_{t3}^{\text{es}} t^3 + \nu_i^{\text{es}} \right) - \frac{1}{1 + \varepsilon} \ln(\lambda (1 - \tau)).
\]

**B.5 Details: Parent’s Problem**

The parent’s problem begins when the parent turns 20 years old. Each year the parent receives income and makes consumption/saving decisions. We assume that all parents make transfers to their children at the year which corresponds to \( t = 1 \) for the child and an age of 43 for the parent. Parents start the model with 0 assets and live until age 65.

For all years when the transfer is not given, the parent simply chooses how much to consume and save. Let \( V_t^P \) denote the parent’s value function in year \( t \). We can write this as

\[
V_t^P (X, I, a_t^P) = \max_c \left[ \frac{c^{1-\gamma}}{1-\gamma} + \beta V_{t+1}^P (X, I, a_{t+1}^P) \right],
\]

subject to:

\[
c = y_t^P + (1 + r) a_t^P - a_{t+1}^P
\]

where \( a_t^P \) is the parent’s assets in year \( t \) and \( y_t^P \) is the parent’s income in year \( t \).

In the year of the transfer, the parent also receives utility from transfers. In this year, we write the parent’s Bellman equation as

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\(^{65}\)This will correspond to age 18 of the child if the parent gave birth to the child at age 25. This is the median age a mother gave birth to their child in the NLSY97.
\[ V_t^P (X, I, a_t^P) = \max_{c, tr^{hs}, tr^{col}} \left[ \frac{c^{1-\gamma}}{1-\gamma} + F (tr^{hs}, tr^{col}, X, I) + \beta E \left[ V_{t+1}^P (X, I, a_{t+1}^P - tr^e) \right] \right] \]

subject to:

\[ c = y_{it}^P + (1 + r) a_{it}^P - a_{it+1}^P. \]

where \( tr^{hs} \) and \( tr^{col} \) are the transfers offered conditional on the child’s education choice, and \( tr^e \) are the realized transfers. As the parent must commit to transfers before the child’s college preference shock is realized, the child’s college choice and therefore the value of \( tr^e \) is stochastic at the time the parent chooses the transfer in the eyes of the parent. \( F (tr^{hs}, tr^{col}, X, I) \) is the expected utility the parent receives from the transfer schedule \( tr^{hs}, tr^{col} \) and is defined in the main text.

**Parent’s Earnings Profile Calibration** We assume that parental earnings are determined by a similar process to the child’s earnings. Specifically, parental earnings are given by

\[ \forall e = H, G: \log y_t^P = \beta_{t1}^{\text{ParEdu}} \text{ParAge}_t + \beta_{t2}^{\text{ParEdu}} \text{ParAge}_t^2 + \beta_{t3}^{\text{ParEdu}} \text{ParAge}_t^3 + v^P. \]

where \( \text{ParAge}_t \) is the parent’s age in period \( t \). The age coefficients, \( \beta_{t1}^{\text{ParEdu}}, \beta_{t2}^{\text{ParEdu}}, \) and \( \beta_{t3}^{\text{ParEdu}} \) are taken from the child’s earnings regression. We assume that the parent’s age coefficients are given by the college age coefficients if at least one parent has attended college, otherwise the parent’s age coefficients are given by the age coefficients for a child that has not attended college.

The term \( v^P \) represents persistent, idiosyncratic differences in earnings across parents. We assume that we observe the parental income variable \( I \) when parents are 40 years old. Therefore, we must have \( y_{40} = I \) for each parent we observe in data. We therefore choose \( v^P \) such that the predicted parental income at age 40 is equal to the observed parental income variable \( I \). We can write this as

\[ v^P = \log I - \left( \beta_{t1}^{\text{ParEdu}} \text{ParAge}_t + \beta_{t2}^{\text{ParEdu}} \text{ParAge}_t^2 + \beta_{t3}^{\text{ParEdu}} \text{ParAge}_t^3 \right). \]
B.6 Enrollment by Gender

B.7 Earnings Profiles Model

B.8 Calibration: Endogenous Ability

For computational simplicity, we do not allow financial aid to depend on merit in this section.

Initial ability, $\theta_0$, is unobserved to the econometrician. So we assume that $\theta$ is distributed as:

$$\ln \theta_0 = \beta \ln I + \varepsilon$$

where $\varepsilon$ is normally distributed. We choose $\beta$ and the variance of $\varepsilon$ to match the variance of log childhood ability and covariance of log childhood ability and log parental income from Agostinelli and Wiswall (2016).
We re-normalize our measure of $\theta$ to match the mean and variance of the measure of final log cognitive skill from Agostinelli and Wiswall (2016). The remaining parameters to calibrate are:

1. $A$: TFP of parental production function.
2. $\gamma_1$: weight on initial ability
3. $\gamma_2$: weight on parental investment
4. $\gamma_3$: interaction term
5. $\sigma^\iota$: variance of $\iota$.

We set $\gamma_1$ equal to the product of the coefficients on lagged ability from Agostinelli and Wiswall (2016) $\gamma_{1,1}\gamma_{1,2}\gamma_{1,3}\gamma_{1,4} \approx 2$. This approximation will be true if the terms on the interaction terms in Agostinelli and Wiswall (2016) are close to zero in all years after the first year.

Then we have four parameters, $A$, $\gamma_2$, $\sigma^\iota$, and $\gamma_3$. We choose these parameters to match the four following moments:

1. Mean of $\theta$
2. Variance of $\theta$
3. Covariance of $\theta$ and parental income $I$.
4. From Agostinelli and Wiswall (2016): The effect on realized years of schooling of a monetary transfer to parents is roughly ten times larger for parents in the 10th percentile of the income distribution compared to those in the 90th percentile.

Loosely speaking, the covariance of $\theta$ and $I$ helps to pin down the importance of parental monetary investments $\gamma_2$. The variance of $\theta$ helps to pin down the variance to shock of ability production, $\sigma^\iota$. The differential effect of monetary transfers for rich and poor parents helps to pin down the interaction between parental investment and initial ability, $\gamma_3$. Finally, the average ability level helps to discipline the TFP of the production function, $\gamma_1$.

The parents problem can therefore be written as:

$$\max_{Invest} \mathbb{E}_i \left[ \tilde{V} \left( \theta_i, Invest, \iota, a_{0}^P - Invest \right) \right]$$

where the value function $\tilde{V} \left( \theta, a_{0}^P - Invest \right)$ is the parents value function and $a_{0}^P$ is the parent’s assets at the beginning of the model. For simplicity, we assume that grants are only a function of income when solving the model with endogenous ability. This considerable simplifies the model solution.
B.9 Varying Borrowing Constraints

To get a sense of how varying borrowing constraints would affect our main conclusions, we have re-estimated a version of the model in which the borrowing limit depends on parental resources.

Here, it was very hard for us to get guidance on what would be a reasonable way to have exogenous borrowing constraints depend on parental income and ability of the child. Hence, we have decided to report a very simple and transparent case in the paper: we assume that children whose both parents have a college degree can borrow twice the amount of the Stafford loan limit. Admittedly, this is ad-hoc in two ways. The first ad-hoc decision is to separate children along the parental education dimension. Our motivation was that parental education strongly correlates with both parental earnings and child’s ability. The second ad-hoc decision we faced was: how much more can these children with highly educated parents borrow? We here decided to just double the amount in the case that we report.

The optimal utilitarian financial aid with parental education dependent borrowing constraints are shown in Figure 20. The shape is slightly different from the baseline optimal schedule, as changes in the borrowing constraints lead to changes in the distribution of the marginal social welfare weights.\footnote{As we have shown earlier, relaxing borrowing constraints for all students reduces the progressivity of the optimal aid schedule. That force is still present here, as some low income students have two college educated parents. However, this force is partially muted by the fact that parental education is increasing in parental income. As such, the optimal aid schedule here is more progressive than the case with relaxed borrowing constraints for all individuals, but slightly less progressive than the baseline case with equal borrowing constraints for all students.} However, the optimal financial aid is still highly progressive.

Figure 20: Optimal Financial Aid with Parental Education Dependent Borrowing Constraints
B.10 Details: General Equilibrium Wages

We assume identical perfectly competitive firms use CES production functions which combine skilled and unskilled labor. Therefore, wages are determined as a function of the ratio of the total skilled labor to the total unskilled labor.

Let $P_U$ and $P_S$ denote the endogenously determined efficiency wages for unskilled and skilled workers, respectively, where skilled workers are those with a college degree and unskilled workers are high school graduates. We allocate half of college dropouts to each of the skill groups, as is common in the literature (e.g. Card and Lemieux (2001)). Suppose an agent’s wages can be written as the product of her efficiency wage and her quantity of efficiency units of labor supplied: $w_{it} = P_{sk}H_{it}$, where $sk \in \{unskilled, skilled\}$ denotes skill level and $H_{it}$ denotes agent $i$’s level of human capital.\(^{67}\)

We assume perfect competitive labor markets. Production at the representative firm is a CES function combining skilled and unskilled labor:

$$Y = A \left( \lambda S^{(\sigma-1)/\sigma} + (1-\lambda) U^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

where $A$ is total factor productivity, $\lambda$ is the factor intensity of skilled labor, and $\sigma$ is the elasticity of substitution between skilled and unskilled labor. We assume $\sigma = 2$. $S$ and $U$ represent the total amount of human capital units supplied by skilled and unskilled workers. We assume the economy is in a long run steady-state equilibrium, and that the economy consists of identical overlapping cohorts. Therefore, as cohorts are identical, the total labor supply in the steady-state equilibrium is equal to the total amount of labor supplied over the life-cycle for a given cohort.

Therefore, we can write:

$$S = \sum_i \sum_t H_{it} \ell_{it} I(ski = skilled)$$

and

$$U = \sum_i \sum_t H_{it} \ell_{it} I(ski = unskilled)$$

Efficiency wages are given by the first order conditions of the firm’s profit maximization problem:

\(^{67}\)We normalize units of human capital such that $H_{it} = 1$ is an efficiency unit of labor is defined as the labor supplied by a male worker whose log wages at age 18 are equal to the constant of the wage equation. Therefore, the constants of the wage functions for skilled and unskilled workers are equal to the logs of the efficiency wages for skilled and unskilled workers.
\[ P^S = A \left( \lambda S^{(\sigma-1)/\sigma} + (1 - \lambda) U^{(\sigma-1)/\sigma} \right)^{1/(\sigma-1)} \lambda S^{-1/\sigma} \]

and

\[ P^U = A \left( \lambda S^{(\sigma-1)/\sigma} + (1 - \lambda) U^{(\sigma-1)/\sigma} \right)^{1/(\sigma-1)} (1 - \lambda) U^{-1/\sigma}. \]

These two functions determine wages endogenously as functions of labor supply.
References


