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Abstract

Formal language rights, such as the status as an official, working, national, or regional language, are normally conferred on a language in a well-defined territory, be it a county, a province or an entire country. For a minority language in a federal state, the status as a rule varies from one jurisdiction to another, depending on the number of speakers in the various federal parts of the state.

We provide a framework for a normative analysis of this type of language policy. The analysis is based on cost-benefit analysis. We first define and discuss the benefit side assuming benefits to be basically proportional to the number of beneficiaries of a given planning measure. We then argue that the costs of a planning measure can be described as a function of two variables “number of beneficiaries” and “size of area of application”. The form of this function characterizes different types of planning measures. This lets us analyze different planning measures in a two-dimensional model characterized by the number of beneficiaries and their geographical distribution in the jurisdiction under consideration.

In praxis simple planning rules have to be used in deciding on the implementation planning measures. The cost structure – as well as the size of the costs of given planning measures – determines how different planning rules function. In addition, also for practical purposes, only a small set of policy categories is tractable. This leads to a multi-stage optimization problem. For any possible category – collection of planning measures – an optimal planning rule has to be found. Then policies consisting of different categories have to be designed. Finally, a policy maximizing social value considering administrative costs is characterized.

1 INTRODUCTION

In the literature on language planning and policy the demand side, or the “need” for public policy, is well developed.¹ The “supply” side – the costs of language policy –, on the other hand, is

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¹ See, for instance, GAZZOLA (2014), GAZZOLA and GRIN (2017), WICKSTRÖM, TEMPLIN, and GAZZOLA (2018), or GRIN (2003), and the references in those contributions. In a more popular vein some of the main arguments can also be found in WICKSTRÖM, GAZZOLA, and TEMPLIN (2018a).

often neglected.² This essay is an attempt at a systematic discussion of a rational language policy when the *per capita* benefits are well defined, and the implementation costs vary and play a decisive rôle for the evaluation of the policy. We argue that the size of costs as well as the structure of costs are crucial for the design of a sensible language policy.

The methodology of the essay takes its point of departure in traditional cost-benefit analysis, which can be modified in order to take account of (re)distributional sides of the policy to be implemented. The benefits of different language-policy measures are in practice determined by the political process and can, hence, be seen as a subjective variable fixed by the policy maker – the “planner”. The costs, on the other hand, are as a rule objectively measurable. We argue that the costs of each individual planning measure can be characterized by their dependence on the number of beneficiaries of the measure and on the geographical distribution of those beneficiaries. A first-best policy with respect to the politically determined benefits would then, for each planning measure, consist of a comparison of these benefits with the implementation costs resulting in a decision to implement the measure if the net benefits (benefits minus costs) are sufficiently high. This, however, would neglect the administrative costs which could become prohibitively high. In order to reduce the administrative costs, rules of thumb for the implementation decisions are required. That is, approximations are necessary for the implementation decision related to most single planning measures. This, however, is not enough; a different rule for each relevant planning measure would also make the language policy too onerous. A limited number of rules, each applied to a set of measures, is necessary to make the policy tractable.

The problem of structuring the language policy then becomes one of associating different planning measures with an appropriate rule. For each rule a policy category consisting of a set of measures is to be defined to which the rule is applied: an optimal structuring of the policy will consist of a number of policy categories with the planning measures in each category characterized by similar benefits, costs, and cost structures; each policy category will have its own decision criterion for the acceptance and implementation of all measures in the category.

The flexibility of having several policy categories in language planning is bound to improve the planner’s goal fulfillment in comparison to a situation with only one category like “official” language. Instead of one decision criterion that is a compromise designed for a great number of planning measures that are very different with respect to their costs and benefits, we should have several different criteria, each designed for a group of measures with similar “supply” technologies and “needs”. However, in an optimal policy the number of categories will be limited by the administrative costs giving rise to a trade-off between better goal fulfillment and administrative ease.

The design of an optimal language policy, hence, has to be done in several steps. First, the general conditions for implementing a single planning measure have to be characterized; second, approximate, sufficiently simple decision rules that can be applied to any measure have to be found; third, the consequences of such rules equally applied to many measures – a planning category – have to be analyzed; fourth, an optimal distribution of the measures on different categories for given decision rules should be determined; and, fifth, the optimal number of categories, given the administrative costs has to be fixed.

The essay is organized as follows: in section 2 the basic approach is discussed, and the

² For a notable exception, see the work of François Vaillancourt and coauthors, for instance, VAILLANCOURT (1997), DESGAGNÉ and VAILLANCOURT (2016), or VAILLANCOURT (2018). A popular treatment can be found in WICKSTRÖM, GAZZOLA, and TEMPLIN (2018b).

general terminology is presented. Section 3 introduces the general framework of the analysis and discusses the implications of various decision rules. In section 4 this analysis is reversed, and the question is asked, how rules should be designed in order to maximize social welfare, however defined. Section 5 draws some general conclusions for practical language policy. The discussion in the main part of the paper is verbal without any formal analysis and should be accessible to anyone with a basic training in academic reasoning. A stringent, slightly formal analysis is relegated to appendices.

2 SETTING THE STAGE: AN ECONOMICS APPROACH TO LANGUAGE PLANNING AND POLICY

Efficiency is a key concept in economics. In any normative analysis, allocative efficiency plays a central rôle as a guide to choosing between possible alternatives. One could say that efficiency tells us how to distribute goods and services in a desirable manner between people in society irrespective of the effects this might have on – explicit or implicate – income or wealth distributions. The main thrust is that there should be no waste. Inputs should be used where they are most productive in the sense of goal fulfillment and goods should be consumed by individuals who value the consumption the most according to given value criteria. Economists talk about Pareto efficiency.³

However, not only allocation (dividing the goods between various functions), but also distribution and redistribution (distributing wealth or income between individuals) are important in normative analysis. Analytically one can treat allocation and distribution separately.⁴ In this essay, we will concentrate on the allocation, or efficiency, aspect, applying a cost-benefit approach to language policy.⁵ In section 4, we will, however, extend the analysis somewhat in order to show how distributional aspects can be taken into account.⁶

2.1 COST-BENEFIT ANALYSIS

Cost-benefit analysis is a standard analytic tool that has been developed for analyzing the efficiency of public projects. It compares aggregated costs and benefits of any proposed planning measure and rates the measure as desirable if benefits exceed costs and as undesirable in the opposite case. A prerequisite for this is that we know costs and benefits. Generally, costs are readily identified and empirically measurable.

Considerable conceptual and empirical problems are connected with measuring the benefits, however. Ideally, we would like to know what each planning measure considered is worth to each concerned individual, also known as the individuals' *propensities to pay* for the measure. On the one hand, this is a conceptual problem, since the notion "propensity to pay" is not well-defined. Due to income effects of the planning measure, what is known as the Scitovsky paradox

³ See any standard textbook, for example NG (2004).

⁴ This is the traditional public-economics analysis, going back to MUSGRAVE (1956/1957). The Public-choice school challenges the relevance of this, noting that because of institutional constraints in reality all efficiency-promoting decisions also have distributional consequences. See, for instance, BUCHANAN and TULLOCK (1962) or BUCHANAN (1987). However, for the purpose of constructing an analytical framework, the analytical separation is useful and productive.

⁵ See also WICKSTRÖM (2016).

⁶ See also the analysis in WICKSTRÖM, TEMPLIN, and GAZZOLA (2018).

can become relevant.⁷ On the other hand, it is also an empirical problem, since measurements of propensities to pay have to be based on information provided by the concerned individuals. They, however, generally have neither incentives to reveal this information truthfully nor the cognitive experience necessary to evaluate the personal consequences of most possible policy measures.⁸

2.2 BENEFITS IN PRACTICE

Hence, objective evaluations of the benefits are not easy to find. This is true about any public policy measure, be it defense spending or investment in road quality. What the public-sector decision makers can do, is to fix the benefits administratively based on their perception of individual valuation, which, at least partially, finds an expression in the political behavior of the beneficiaries as voters. We will therefore take the benefits of various planning measures as an exogenous, politically or administratively determined variable. The value of this variable, of course, also reflects the political ideology of the public planner and can vary widely from one country to another or from one government to another in the same country. Our analysis cannot judge whether the politically determined size of the benefits are “correct” in some sense; it only shows how to conduct a rational policy once the size of the benefits has been fixed.

The goals of the planner can include positive or negative “discrimination” of various kinds. Some possible objectives are:

- to reflect the propensities to pay of the beneficiaries for each policy measure as objectively as possible (the benchmark case)
- to exaggerate the propensities to pay of the members of an autochtone minority (preservation of traditional minorities)
- to downplay the propensities to pay of the speakers of the language of an earlier colonial power (nation building)
- to curtail the propensities to pay of the speakers of a language whose speakers earlier constituted the dominant political power (reversal of historical injustices)
- to underestimate the propensities to pay of speakers of immigrant languages (accelerating integration)

It is easy to find examples of each objective in the real world; one can also readily extend the list with many other, more or less noble, objectives of the policy maker.

2.3 NOTATION, TERMINOLOGY, AND ANALYTICAL AGENDA

We generally limit ourselves to a given jurisdiction and define a *language-planning measure* as the smallest unit of language policy; we use an index i to denote it.⁹ The costs of implementing measure i are χ^i , and the number of potential beneficiaries of the measure is n . The spatial

⁷ See SCITOVSKY (1941). For a lucid discussion of the general problem of consumer surplus and the associated concept of propensity to pay, see MOREY (1984).

⁸ Cf. GINSBURGH (2017).

⁹ For some examples, see the discussion of table 3.1.

size (area) of the jurisdiction analyzed is a . The planning measures are bunched together into *planning categories*, denoted by index b , each category consisting of one or more measures. A planning measure i belongs to one and only one category b .¹⁰ A *language policy* \mathfrak{P} is defined as a collection of categories. A language policy is assumed to contain all possible planning measures, each belonging to some (one and only) category.¹¹ In other words, the language policy considers the introduction of a large number of individual planning measures. These are divided into different planning categories. The decision making for the language policy then involves both deciding which measures belong together in which category and finding criteria for judging a category desirable or not. We begin by evaluating an individual measure, then different possible categories, and finally the entire policy. In each step, there are trade-offs to consider.

The point of departure of the analysis is, hence, the single measure and the desirability of its implementation. The average *per capita* benefit (expressed in monetary terms) of measure i is, as noted above, determined by the planner. It is written as β^i and assumed to be independent of n . In the eyes of the planner, the net benefit to society, denoted by u^i , of implementing measure i is then the difference between gross benefits and the implementation costs of the measure:

$$u^i := \beta^i n - \chi^i \quad (2.2)$$

The planner takes the size of u^i as the basis for evaluating planning measure i . This makes comparisons of different measures and combinations of measures possible, as well as the setting of priorities.

3 THE GENERAL MODEL

We focus on the structure of costs. For each planning measure we require that the resulting goods be available to each beneficiary in the same way and quality and then describe the costs as a function of the number of beneficiaries n and the area of application of the measure a . Goods are characterized by different properties. One such property is the degree of joint consumption that is possible. This ranges from perfectly joint consumption (non-rival goods), like listening to a radio program, to perfectly individual consumption (rival goods), like eating a cake. A perfectly non-rival good will only cause fixed costs relative to the number of beneficiaries (that is, the costs are independent of the number of people consuming the good) and a perfectly rival good will give rise to a proportional (or affine, if there are fixed costs) cost structure in the number of beneficiaries. A good displaying less than perfect non-rivalry (positive economies of scale in consumption) will lead to a concave cost structure.¹²

In table 3.1, we have collected the four extreme cases of cost structures for language planning measures in dependence of the number of beneficiaries and the size of the territory where the

¹⁰ Formally, let the set of all language-planning measures be denoted by I and the set of planning measures in category b by $I^b \subset I$. Then, if $i \in I^b$ and $b \neq b'$, $i \notin I^{b'}$.

¹¹ Formally we write:

$$\bigcup_{I^b \in \mathfrak{P}} I^b = I \text{ and } I^b \cap I^{b'} = \emptyset \text{ for } b \neq b' \quad (2.1)$$

¹² For a more detailed discussion of the various types of goods resulting from language planning the reader is referred to WICKSTRÖM, TEMPLIN, and GAZZOLA (2018).

The costs of the realization of a measure	<i>are independent of the size of the territory (a)</i> $\sigma = 0$	<i>increase proportionally with the size of the territory (a)</i> $\sigma = 1$
<i>are independent of the number of individuals (n)</i> $\rho = 0$	non-spatial and non-rivaling good $(\rho, \sigma) = (0, 0)$	spatial and non-rivaling good $(\rho, \sigma) = (0, 1)$
<i>increase proportionally with the number of individuals (n)</i> $\rho = 1$	non-spatial and rivaling good $(\rho, \sigma) = (1, 0)$	spatial and rivaling good $(\rho, \sigma) = (1, 1)$

TABLE 3.1 A classification of language-planning measures according to the type of the realized good

measure is being implemented. For short we use two parameters, ρ and σ , to describe the dependency of costs on the number of beneficiaries and on the size of the area of implementation. An example of a measure producing a non-rival and non-spatial good, $(\rho, \sigma) = (0, 0)$, would be status planning requiring the use of the language in question in official documents like in passports, on money bills, or in public decrees. A spatial and non-rivaling good, $(\rho, \sigma) = (0, 1)$, would be street signs with street names in a minority language. A non-spatial and rivaling good, $(\rho, \sigma) = (1, 0)$, would be the use of a minority language in the prime minister's call center where the citizens can inform themselves about the official language policy of the government. Finally, public education in a minority language comes close to a spatial and rivaling good $(\rho, \sigma) = (1, 1)$.¹³ In general, of course, any intermediate cases are possible with various degrees of economies of scale in the number of beneficiaries (n) and the size of the territory (a). In appendix A, we provide a parametric specification that is used to calculate the following diagrams. In the appendix, the notation is simplified and a new – for the analysis more relevant – variable, the density of the minority-language speakers δ in the concerned jurisdiction is introduced instead of a :

$$\delta := \frac{n}{a} \quad (3.1)$$

The net benefits of measure i are then determined by the size of the minority population (n) and its density (δ) in the jurisdiction considered:

$$u^i = \beta^i n - c^i(n, \delta), \quad (3.2)$$

where the costs of measure i as a function of n and δ are denoted by c^i . Sometimes it is convenient to divide by β^i defining:

$$g^i(n, \delta) := n - \frac{c^i(n, \delta)}{\beta^i} \quad (3.3)$$

¹³ Of course, important is only the net costs of adding information or education in an additional language, after subtracting the savings due to the reduction in the demand for services in alternative languages.

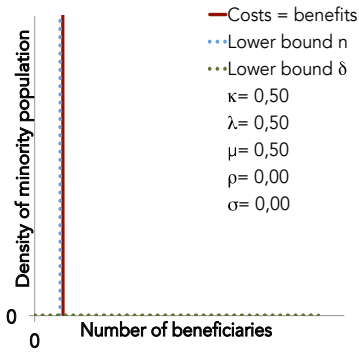


FIGURE 3.1 Efficient rights allocations with a non-rival, non-spatial good

The function g^i can be seen as representing the cost structure of the planning measure, whereas β^i represents its importance. For the further analysis this separation of the net benefits will prove to be useful.

3.1 THE EFFICIENCY FRONTIER

If the n and δ of the jurisdiction under consideration are such that $g^i(n, \delta) > 0$, benefits of measure i exceed its implementation costs and planning measure i gives rise to net benefits. On the other hand, if $g^i(n, \delta) < 0$, the benefits are not high enough to cover the costs and the planning measure leads to net costs for society. Hence, the solution of the equation $g^i(n, \delta) = 0$ gives us the combinations of values of n and δ of a jurisdiction for which the benefits of the planning measure exactly balance the costs. We talk about the efficiency frontier. If g^i does not depend on δ , the efficiency frontier is a critical value n^{i*} (which could be infinite if g^i is negative for all values of n) with the property that planning measure i is efficient if $n \geq n^{i*}$ and inefficient if $n < n^{i*}$. Similarly, if g^i is independent of n , we obtain a critical value δ^{i*} . In general, however, we obtain an efficiency frontier involving both n and δ . In appendix B it is shown that the frontier as a curve in the $(n - \delta)$ -space has a non-positive slope. The area above the curve contains the combinations of n and δ characterizing jurisdictions in which the measure produces net benefits – it is efficient:

PROPOSITION 1 The efficiency frontier of a planning measure is either a constant value of n or of δ or it displays a trade-off between the number of beneficiaries of the planning measure n and their spatial density δ in the jurisdiction under consideration.

In the following, we illustrate the efficiency frontier for different dependencies of the costs on the number of beneficiaries (for different values of ρ) or on the area of implementation (for different values of σ). For the definition of the other parameters listed in the diagrams, the reader is referred to appendix A.

3.1.1 Values of ρ and σ equal to zero or one

We first find the efficiency frontiers in the four extreme cases of table 3.1:

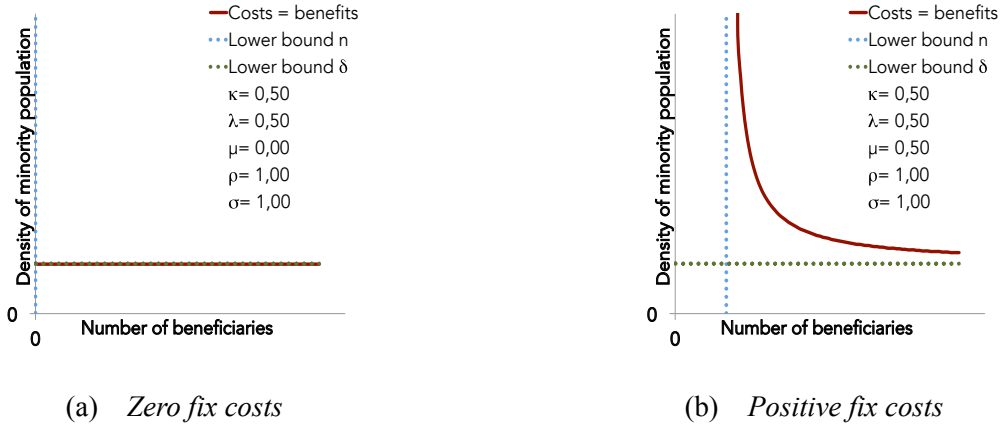


FIGURE 3.2 Efficient rights allocations with a rival, spatial good

3.1.1.1 Case $(\rho, \sigma) = (0, 0)$

The benefit-cost expression 3.3 becomes $g(n, \delta) = n - \mu$ and the efficiency condition is satisfied if and only if n reaches or exceeds a critical value n^* :

$$n \geq n^* := \mu \quad (3.4)$$

Of course, if the fixed costs (μ) are zero, the measure is cost-less and always efficient, as long as the benefits are positive ($\beta > 0$).

The case is illustrated in figure 3.1.

3.1.1.2 Case $(\rho, \sigma) = (1, 1)$

Expression 3.3 now gives us a critical value of δ , δ^* , if there are no fixed costs, and a downward sloping curve if there are positive ones.

The case is illustrated in figures 3.2a for zero fixed costs and in 3.2b for positive fixed costs.

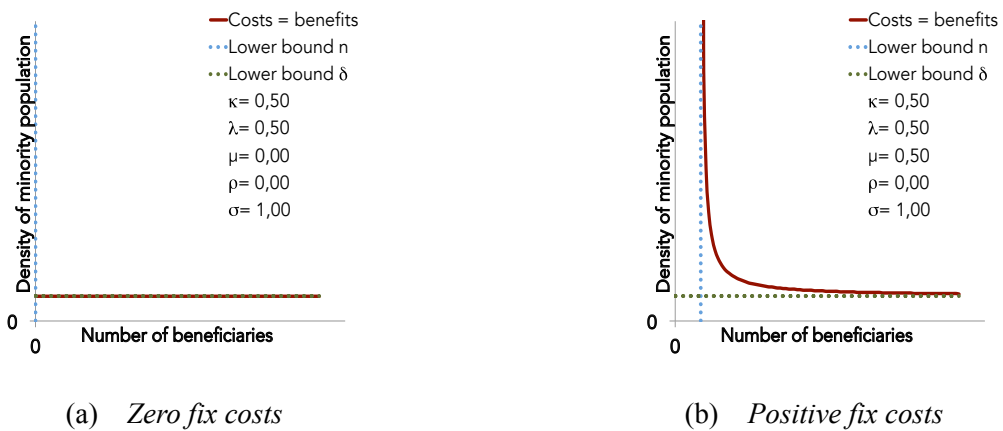


FIGURE 3.3 Efficient rights allocations with a non-rival, spatial good

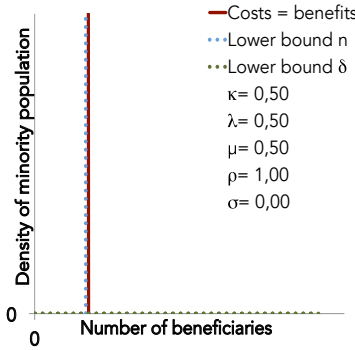


FIGURE 3.4 Efficient rights allocations with a rival, non-spatial good

3.1.1.3 Case $(\rho, \sigma) = (0, 1)$

Again, without fix costs efficiency is determined by a critical value of the population density. With fix costs the efficiency frontier is again a falling curve in the $(n - \delta)$ -diagram.

The case is illustrated in figures 3.3a for zero fixed costs and 3.3b for positive ones.

3.1.1.4 Case $(\rho, \sigma) = (1, 0)$

The efficiency frontier is given by a critical value of n that is equal to zero in the absence of fixed costs and positive if the fixed costs are positive.

This is illustrated in figure 3.4.

3.1.1.5 Algebraic expressions for the efficiency frontier

These results indicate that – in dependency of the structure of the costs – different decision criteria for the introduction of language planning measures are called for. We have collected the values defining the efficiency frontier for the given measure in table 3.2. An efficient decision would in each case mean that the measure should be implemented in jurisdictions with sufficiently high values of the size of the minority population and/or of its density in the territory.¹⁴

For these corner cases, we get a critical-value criterion in all but the two cases marked in

(ρ, σ)	$\mu = 0$	$\mu > 0$
$(0, 0)$	–	$n^* := \mu$
$(0, 1)$	$\delta^* = \lambda$	$\delta^* = \frac{\lambda}{1 - \mu/n^*}$
$(1, 0)$	$n^* = 0$	$n^* = \frac{\mu}{1 - \kappa}$
$(1, 1)$	$\delta^* = \frac{\lambda}{1 - \kappa}$	$\delta^* = \frac{\lambda}{1 - \kappa - \mu/n^*}$

TABLE 3.2 Efficiency frontiers for different language-planning measures

¹⁴ Formally, $(n, \delta) \geq (n^*, \delta^*)$.

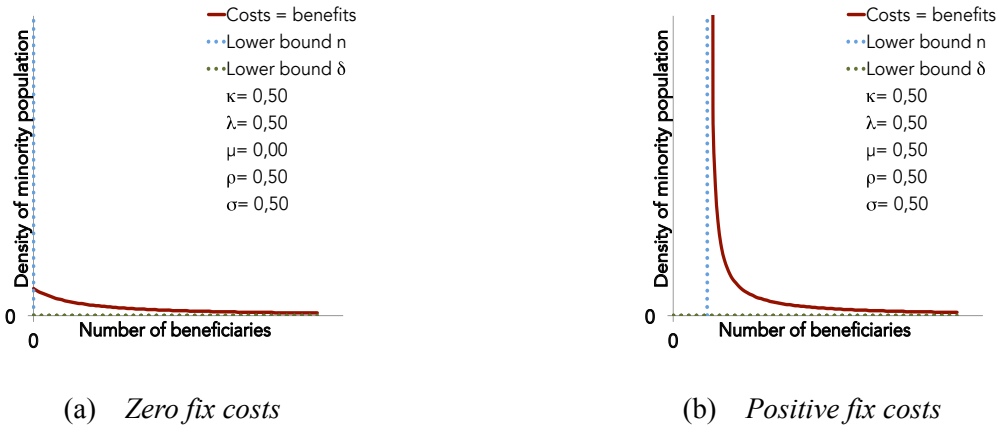


FIGURE 3.5 Efficient rights allocations with a partially rival and partially spatial good

bold type in the table. However, dependent on the type of good, there is a critical value of either the size of the minority population or the density thereof. It is interesting to note that in no case the most common decision rule in minority-language law, a percentage criterion is called for. In the two cases without a clear critical number, we have to look for more complicated criteria. This is discussed in section 3.3.2 below.

3.1.2 Values of ρ and σ between zero and one

Looking at intermediate cases between those in table 3.1, we, in general, find a situation similar to figures 3.2b and 3.3b. However, a critical-value criterion for n , written n^* , always obtains if the costs, $\chi(n, a)$, are independent of a . The value of n^* is possibly infinite.

As an example, we in figures 3.5a and 3.5b show the efficiency frontier without and with fix costs for the case when the resulting goods are partially rivaling and partially spatial. In figures 3.6a and 3.6b, as well as 3.7a and 3.7b the efficiency frontiers for almost non-rival and non-spatial as well as for almost perfectly rival and spatial goods are shown.

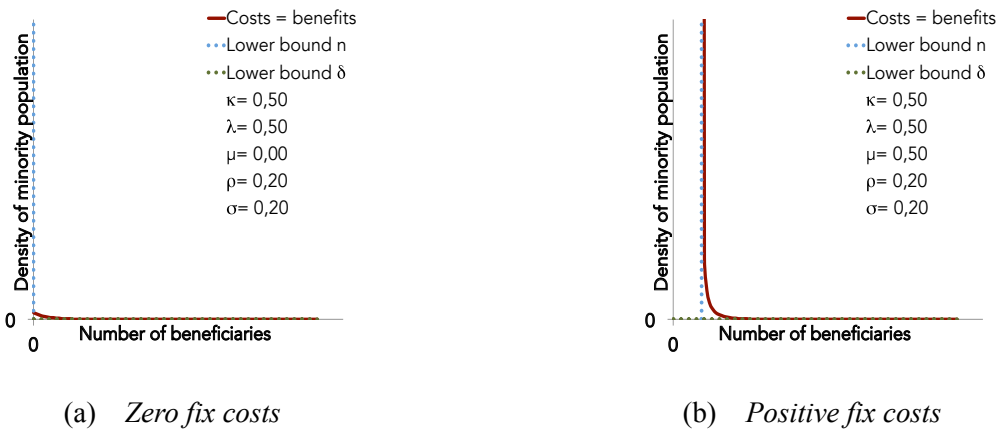


FIGURE 3.6 Efficient rights allocations with an almost non-rival and almost non-spatial good

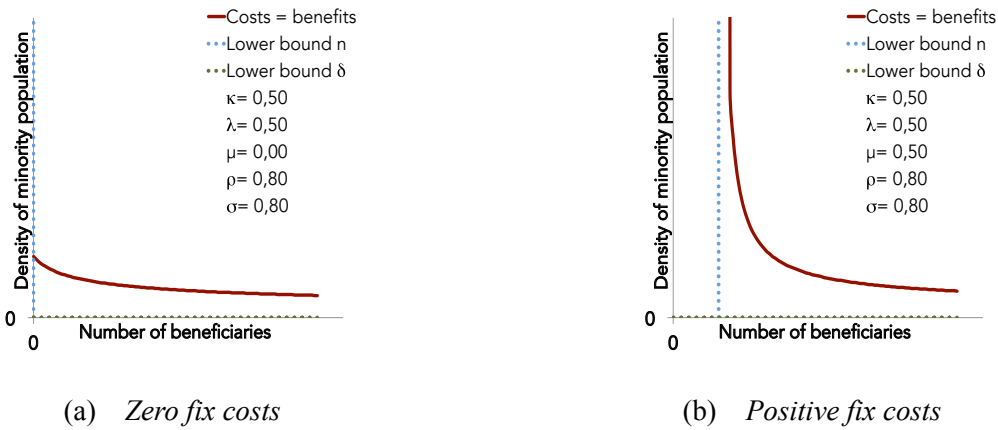


FIGURE 3.7 Efficient rights allocations with an almost rival and almost spatial good

The lower bounds on the variables n and δ are given by some values n^{**} and δ^{**} . If n or δ lay below one of these values, the planning measure cannot be efficient. The lower bounds for our specification of the cost function are collected in table A.1 in appendix A. We see that unless the fix costs are zero, there is always a positive number of beneficiaries that has to be present in order for the language planning to be efficient, since the value of the benefits at least have to exceed the fix costs.

3.2 CHANGES IN THE POPULATION STRUCTURE

We have identified the relevant population structure of any jurisdiction through the parameters n and δ . This specifies a point in the diagram with the efficiency frontier. If the population structure is to the right or above the efficiency frontier, the planning measure is efficient and should be introduced. The population structure changes with migration and if the size of the relevant territory is altered. Some possible changes are indicated in figures 3.8a through 3.9b.

We first look at migration into the area under consideration. Balanced migration with given proportions of the majority and minority population is illustrated in figure 3.8a. This type of migration will make it more likely that the conditions for introducing a language planning measure will be satisfied, since both the density and the number of beneficiaries increase. More common in reality, however, is an influx of only the majority population while the traditional minority population remains the same. This does not change the *relevant* population structure, since neither the number of beneficiaries, nor the population density of the beneficiaries changes. Although the percentage of the minority population changes, we remain in the same point in the diagram, see figure 3.8b. An immediate consequence of this, is that the fraction of the total population belonging to the minority is irrelevant for a language policy building on a cost-benefit analysis. In other words, a percentage rule cannot lead to efficient results, see also section 3.3.2.1. Abolishing language rights in communities with an increasing total population, but constant minority population, finds no support in an analysis based on cost-benefit analysis. In the case of “new minorities” entering the area, that is speakers of a different language coming as refugees or in search of a better life, the situation of figure 3.8a applies. Both the absolute number (n) and the density of the minority (δ) increases, making the fulfillment of conditions

for introducing language planning more likely.

The effect of joining two jurisdictions together in order to form a bigger unit can be very different depending on the demographic characteristics of the jurisdictions. If the population structure in the two jurisdictions being joined is the same, the density of the beneficiaries does not change but their number is increased. The argument for the language planning measure in question is strengthened; see figure 3.9a. The more common case in the real world, however, is that the minority is concentrated in one of the jurisdictions to be joined. In this case, illustrated in figure 3.9b, the number of beneficiaries remains constant, but their density decreases and less minority rights can be justified. This also implies a decrease in efficiency, since efficient planning measures become inefficient or less efficient when they are implemented over a larger area. Inversely, drawing jurisdiction borders for planning measures according to the habitation patterns of the minority will increase the efficiency of planning measures. Finally, members of the minority population moving into a jurisdiction will make planning measures more efficient, hence justifying more minority rights.¹⁵

We collect the main insight from the discussion in formal statements:

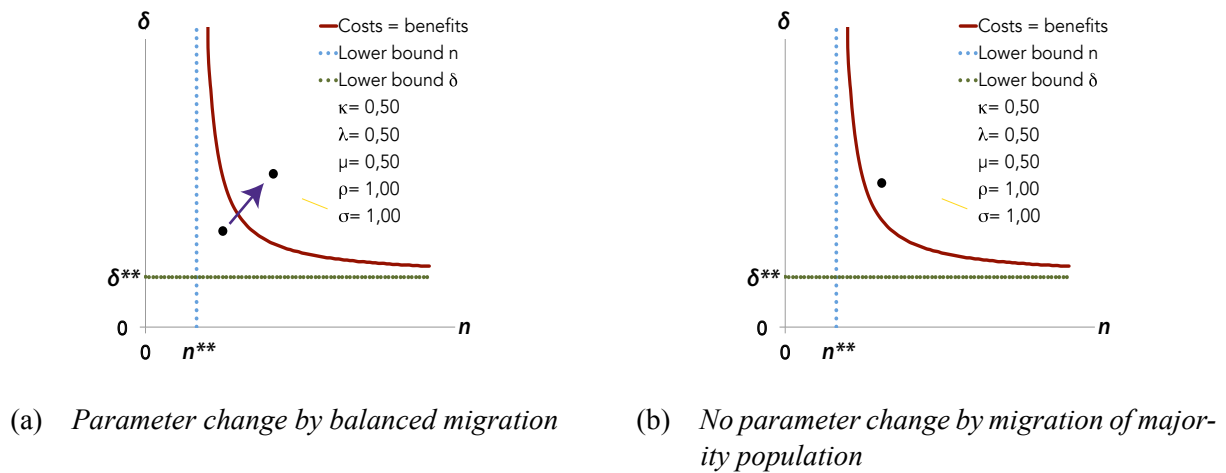
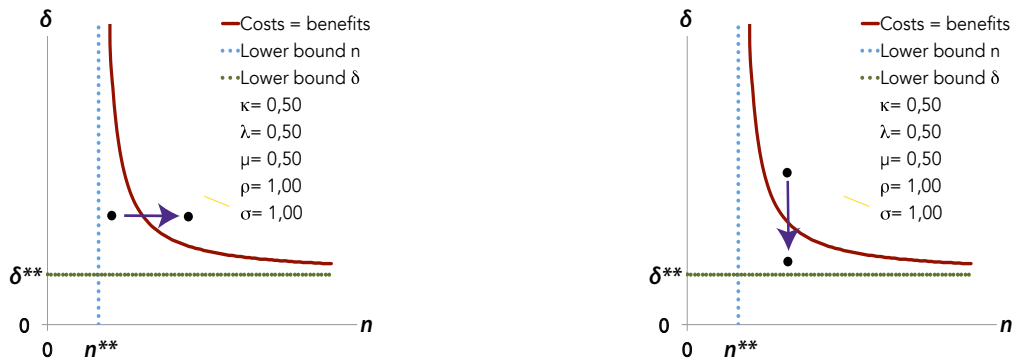


FIGURE 3.8 Effects of migration

¹⁵ A merger of jurisdictions can have a further consequence, not treated in our framework. If the minority population is in the majority in one jurisdiction and in the minority in the merged jurisdiction, it will change from being first language in one area to being the second language everywhere in the post-merger situation. This can have consequences for the use of the minority language as working languages in public institutions and in contacts with the central state. The minority language will in this way be weakened. See the current debate in Norway or Finland, where the plans to merge local jurisdictions into larger (presumably in other respects more efficient) units by many is feared to have such consequences in spite of the fact the formal minority-language rights are not affected.



(a) *Parameter change by a uniform distribution of the minority*

(b) *Parameter change by a regional concentration of the minority*

FIGURE 3.9 Effects of centralization

PROPOSITION 2 For any planning measure, the efficiency frontier divides the jurisdictions into those where the measure is efficient and those where it is inefficient depending on the number of the beneficiaries (n) and their density (δ) in the jurisdiction, such that if the measure is efficient in a jurisdiction, it is also efficient in all jurisdictions with the same or higher values of n and δ and, conversely, if it is inefficient in a jurisdiction, it is also inefficient in all jurisdictions with the same or lower values of the two parameters.

COROLLARY 2A A decision rule based on the fraction of beneficiaries as part of the total population – a “percentage” rule – cannot be an efficient rule.

COROLLARY 2B A migration of the majority population into jurisdictions populated by the minority does not justify any change in minority rights in the concerned jurisdictions.

COROLLARY 2C The creation of bigger jurisdictions through the merger of jurisdictions containing a minority population with jurisdictions without the minority will justify a decrease in minority rights if costs are at least partially spatial. As a result an optimal policy becomes globally less efficient.

COROLLARY 2D A federal structure with jurisdiction borders drawn according to the habitation patterns of minorities makes more efficient minority-rights allocations possible than a federal structure that is independent of those habitation patterns.

COROLLARY 2E If the minority population is concentrated to a limited area of the country, a more efficient implementations of minority rights is possible through the appropriate choice of the number and geographical extension of jurisdictions than in a situation with the minority more or less equally spread out over the entire country.

3.3 RULES

Even if for every planning measure one could find the efficiency frontier and determine whether the minority-population density and the number of beneficiaries lay above or below the efficiency frontier, in most cases it is convenient to have rules-of-thumb for introducing planning measures. Our analysis above clearly implies that such rules would build on a combination of critical values of the two variables, n and δ . The simplest rule would be that both the number of beneficiaries and the minority-population density reach some indicated values. In some cases, the efficiency frontier will be vertical and in some cases horizontal. A correct critical value of n or δ , respectively, will in this case give an exact indicator for the efficiency of a given measure. If the frontier shows a trade-off between n and δ , a combination of critical values can lead to inefficient results. If an efficient planning measure is rejected, we talk of an *exclusion error*,¹⁶ and if an inefficient measure is accepted, of an *inclusion error*.¹⁷

3.3.1 Critical values in only one of the variables

A vertical efficiency frontier obtains if $g^i(n, \delta)$ does not depend on δ . This happens for perfectly non-spatial measures ($\sigma = 0$), and the total size of the minority population has to reach some value n^{i*} for measure i to be efficient.¹⁸

A horizontal efficiency frontier can only occur in the absence of fixed costs ($\mu = 0$) in two rather special cases: for perfectly non-rivaling and perfectly spatial ($\rho = 0, \sigma = 1$) measures as well as for perfectly rivaling and perfectly spatial ($\rho = 1, \sigma = 1$) measures.¹⁹

3.3.2 Approximate rules

For an efficiency frontier displaying a trade-off between the size of the minority population and its population density, the simplest rule, as suggested above, is a two-parameter rule fixing a minimum value of n , n^* , and a minimum value of δ , δ^* , that jointly form a lower limit for the values of n and δ in the jurisdiction considered. If the policy criteria n^* and δ^* are set too high, we risk an exclusion error, the policy being exceedingly restrictive; see figure 3.10a. On the other hand, if the policy is very accommodating with low values of the parameters, we can

¹⁶ In the literature this is traditionally referred to as an error of type I.

¹⁷ In the literature this is traditionally referred to as an error of type II.

¹⁸ In the specification in appendix A, the critical value n^* is given as the solution to

$$n - \kappa [(1 - \rho + n)^\rho - (1 - \rho)^\rho] - \mu = 0 \quad (3.5)$$

if $\rho < 1$. If $\rho = 1$, the critical mass is either $n^* = \mu/(1 - \kappa)$ if $\kappa < 1$ or infinite if $\kappa \geq 1$.

¹⁹ In our specification in appendix A for $\kappa < 1$; in the first case the critical value is $\delta^* = \lambda$ and in the second case it is $\delta^* = \lambda/(1 - \kappa)$.

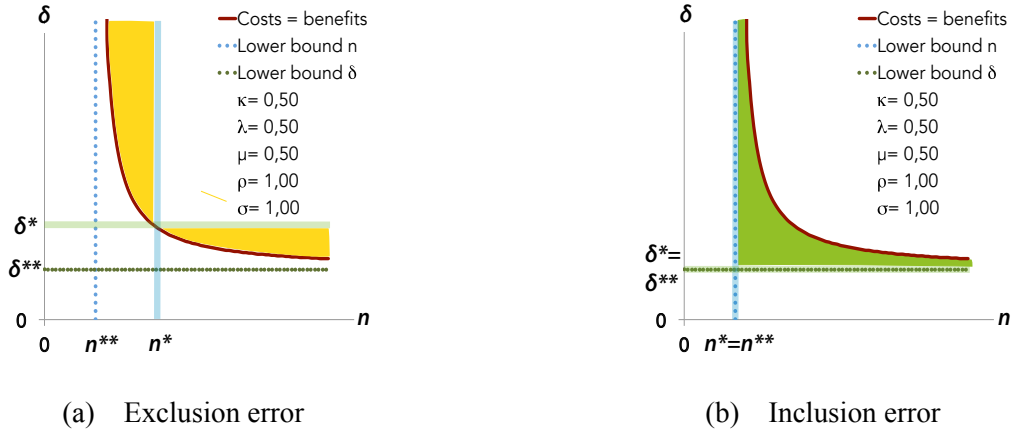


FIGURE 3.10 Types of error by two-parameter language planning

end up with an inclusion error. If the errors occur, of course, depends on whether the values of the parameters in the relevant jurisdictions place the jurisdictions in the error zones. See the discussion below in section 4.

3.3.2.1 Percentage rule

In many countries, a percentage rule is applied in order to grant minority rights.²⁰ The percentage criterion has very different implications in urban and rural areas. Denote the fraction of the minority population by π and the total population in the area under consideration by N . Then $\pi = n/N$. Let $\Delta := N/a$ be the total population density. We then find:

$$\delta = \frac{n}{a} = \frac{n N}{a N} = \frac{n N}{N a} = \pi \Delta \quad (3.6)$$

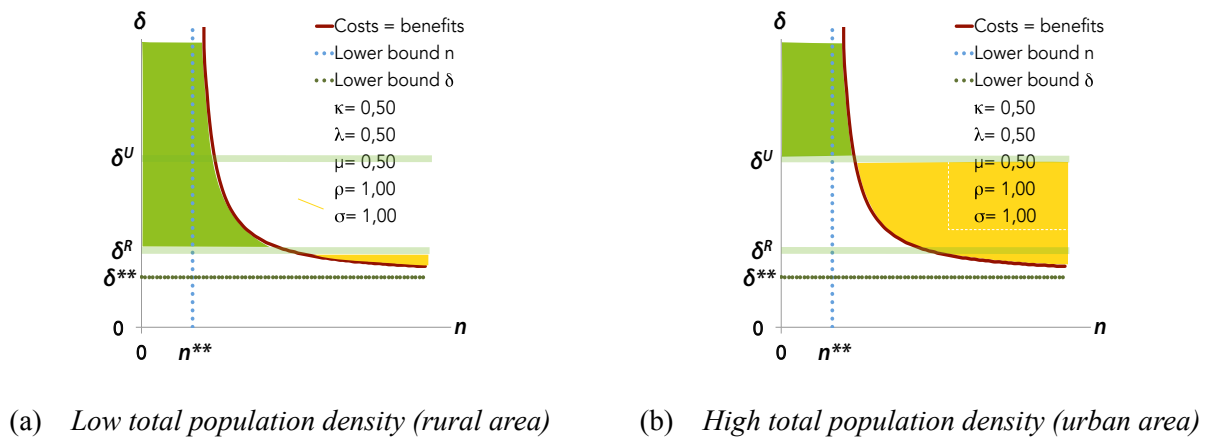
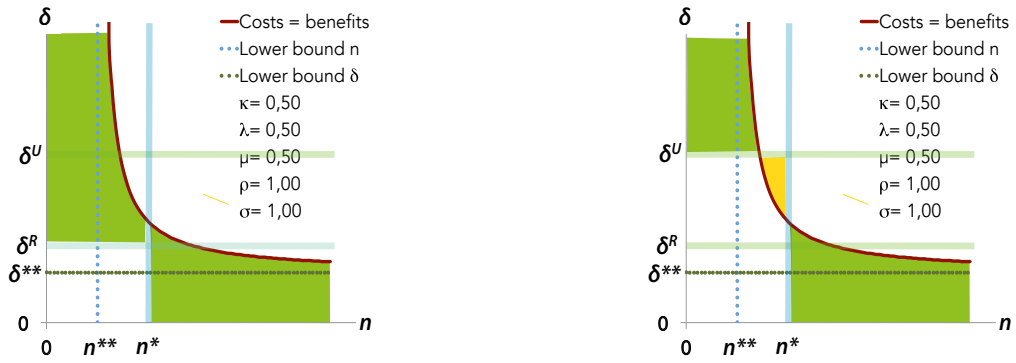


FIGURE 3.11 Errors by percentage-rule language planning

²⁰ Slovakia and Romania, for instance, require a minority to make up at least 20% of the total population in order to grant it certain minority right.



(a) Low total population density (rural area)

(b) High total population density (urban area)

FIGURE 3.12 Errors by “Finnish”-rule language planning

A critical percentage value $100\pi^*$ then implies a critical value of the density of the beneficiaries δ^C :

$$\delta^C = \pi^* \Delta \quad (3.7)$$

Δ varies and is higher in urban areas than in rural areas, $\Delta^U > \Delta^R$. Hence, for a given percentage criterion $100\pi^*$, also δ^C is higher in an urban than in a rural area, $\delta^U > \delta^R$. This is illustrated in figure 3.11.

It is evident that the percentage criterion can lead to a large inclusion error in rural areas and to a large exclusion error in urban areas. That is, the rule can be fairly generous to a rural minority in comparison to an urban population, where a large exclusion error threatens the rights of the minority.

3.3.2.2 The “Finnish” rule

In Finland, a double rule is in effect. Rights are granted if either the minority reaches a certain percentage of the total population (8%) or a certain number of speakers (3 000) in the jurisdiction under consideration. The critical value in the number of beneficiaries changes the structure of errors in comparison to a pure percentage criterion; see figure 3.12. The rule now becomes much more accommodating for the minority population and the occurrence of exclusion errors is considerably reduced, both in the rural and in the urban setting, but it remains larger in the urban situation.

We can conclude:

PROPOSITION 3 A percentage rule tends to benefit rural minorities in comparison to urban minorities. It can be seen as a redistribution in favor of rural minorities at the cost of urban minorities (under the assumption that taxation is the same in rural and urban areas).

COROLLARY 3A If the percentage rule is combined with a critical value for the size of the minority population, the exclusion error is considerably reduced as is the redistribution between urban and rural areas.

3.3.3 General rule

Any combination or function of the parameters n and δ can, of course, serve as a rule for the introduction of a planning measure. This can reduce the size of the exclusion error (like in the case of the “Finnish” rule), the inclusion error, or both. That is, the rule can be made more efficient. This, however, comes at a cost: the rule is more complicated to administer, and we have a trade-off between implementation costs and administrative costs. This trade-off will not be analyzed any further, but the formal side of general rules is discussed in appendix C.

4 OPTIMAL LANGUAGE PLANNING

In the best of all worlds each planning measure is applied wherever its benefits exceed its costs. The value of the measure is the difference between benefits and costs. The total value of the language policy is the aggregation of these values over all measures with non-negative benefit-costs differences. This presupposes that there is full flexibility in the policy, every jurisdiction above the efficiency frontier of every measure will be able to enjoy the measure. This is our benchmark.

In reality, however, this is not possible for several reasons. *First*, the rules and criteria for determining if a given measure should be implemented in a given jurisdiction have to be relatively simple in order to be manageable in practice. This leads to inclusion and exclusion errors. *Second*, we cannot have a separate rule or even a separate decision criterion by a given rule for every possible measure. Measures have to be collected into categories, each category having its own rule and decision criterion. This leads to compromises between different measures which, in turn, will increase the errors of both types. *Third*, there are administrative implementation costs. These costs tend to increase with the number of categories. On the other hand, the exclusion and inclusion errors, tend to decrease with the number of categories. There is, hence, a trade-off between the administrative costs and the implementation costs.

To find the social value of a given language policy, we have to subtract both the administrative and implementation costs from our benchmark (rules identical to the efficiency frontiers for all measures). An optimal policy is a policy which maximizes the social value. Since the benchmark is fixed, we can alternatively minimize the sum of costs due to the implementation errors and the administration of the policy in order to find the optimal policy. That is the approach in this section.

We start out by looking at the implementation costs of a single planning measure applied to different jurisdictions. We then bunch several individual measures into categories and find the corresponding implementation costs. Now, two questions arise: In order to minimize the implementation costs of a planning category, which measures belong together, and how are we to find a decision criterion for the aggregated measures? The answers to these questions turn out to be rather complicated, and we can only give some general characteristics.

Finally, having characterized the implementation costs of all possible categories, we can combine categories into policies. This gives us the implementation costs of any policy as the sum of the implementation costs of the categories making up the policy. To these implementation costs of each possible policy we then have to add the administrative costs of the policy, which we assume to depend on the number of categories in the policy. An optimal policy is then a policy that minimizes the sum of implementation and administrative costs.

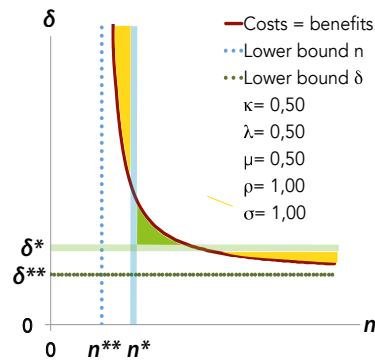


FIGURE 4.1 Minimal errors by two-parameter language planning

The analysis is built up in five stages. After analyzing a single measure in the first stage, in the second one we look for a criterion leading to the smallest aggregate size of the implementation errors (of both types) of any given category of individual planning measures. Then, as a third stage, the size of such aggregate costs for different distributions of the planning measures on a given fixed number of categories is characterized. In the fourth stage, for each fixed number of categories the distribution of measures on the categories giving rise to the lowest total size of implementation costs is found. In the fifth and final stage the number of categories in the policy is varied, the administrative costs are added to the minimal implementation costs, and the total costs of one, two, three etc. categories are found. The optimal policy is then the one giving rise to the smallest total costs.

We discuss the five stages in the following. In order to make the discussion tractable, we exemplify the analysis with a very simple stylized example, which will accompany the discussion. In appendices E, F, G, and H the analysis is made more stringent.

4.1 OPTIMIZATION OF THE DECISION CRITERION FOR A SPECIFIC PLANNING MEASURE

Choosing a critical value of the parameters of any rule, one minimizes the (weighted) sum of the two types of errors. A possible choice is illustrated in figure 4.1 with the rule being a combination of the size of the minority population (n) and its population density (δ) in the relevant jurisdictions. The size of the errors, however, depends on how many jurisdictions are found in the shaded areas or, in general, on the distribution of the parameters n and δ among the jurisdictions under consideration.

As we defined the two types of errors in section 3.3.2, we really only expressed the potentiality of such errors occurring. In order for the errors to really occur, there have to exist relevant jurisdictions to which the criterion is applied, and the jurisdictions have to have values of n and δ placing them in the shaded areas. In figure 4.2, we show two identical situations as far as the costs and benefits of a planning measure are concerned, and we apply the same criterion for the language policy – minimal values of n and δ , n^* and δ^* , which have to be reached in a jurisdiction in order to justify the implementation of the measure. However, the distributions of the characteristics of the jurisdictions are slightly different in the two cases. In figure 4.2a the planning measure will not be chosen by the given criterion although the measure would be efficient in all jurisdictions. We have a clear case with exclusion errors in all jurisdictions. In

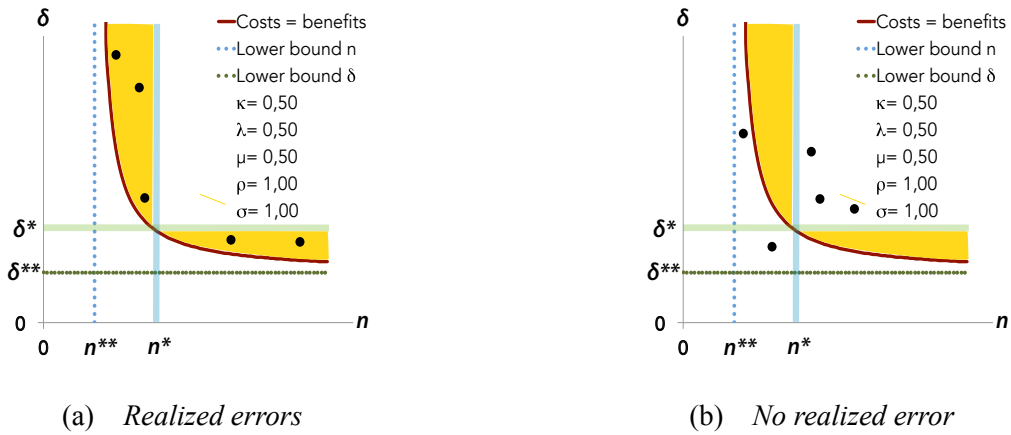


FIGURE 4.2 Exclusion errors in countries with demographically different jurisdictions (each point represents a jurisdiction)

figure 4.2b, on the other hand, the criterion will give efficient results in all jurisdictions, leading to the introduction of the measure in three jurisdictions and to its rejection in two.

Hence, to make rational language planning, the policy maker not only needs to know the efficiency frontier, but also the distribution over the parameters n and δ of the jurisdictions to be subjected to the chosen criterion. In appendix D we describe this distribution with the help of a function specifying the density of jurisdictions in the $(\delta - n)$ -space. A sensible policy objective could be to look for a minimum of the sum of exclusion and inclusion errors. The shaded areas in the diagram would then have to be weighted by the number of jurisdiction with values of δ and n in the shaded areas. The so weighted size of the shaded areas – evaluated in terms of the *per capita* benefits β – would give us the size of the errors.²¹

If the policy maker likes to redistribute in favor of the minority, she should weigh an exclusion error more heavily than an inclusion error. This way, approving a measure that is not efficient is a less severe error than not enacting a measure that is efficient. There would be a bias in the direction of supporting minority rights:²²

PROPOSITION 4 The wish to redistribute in favor of a minority can be realized by weighing exclusion errors more heavily than inclusion errors in the policy objective.

4.2 OPTIMAL CRITERIA AND ERRORS FOR DIFFERNT CATEGORIES OF PLANNING MEASURES

As mentioned above, a practical language policy will have to operate with a limited number of categories. That is, several language-planning measures will have to be collected into one category with one criterion for enacting all the measures in the category. The goal of the planner can be to minimize the (weighted) sum of the two types of error. How this can be done is illustrated in a simple example pictured in figure 4.3. There are four jurisdictions, A, B, C , and D , in the country under consideration with a minority population of size a, c, e , and g , respectively.

²¹ In appendix D we show in more detail how to treat the case with a smooth distribution of jurisdictions over the $(\delta - n)$ -space.

²² This is discussed in more detail in appendix D.3 and section 4.4 below.

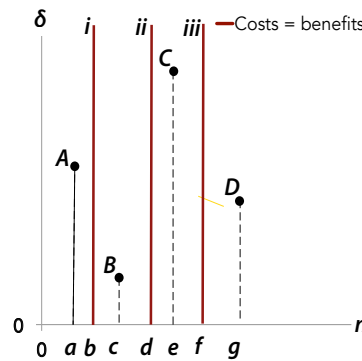


FIGURE 4.3 Three planning measures and four jurisdictions

Three planning measures, called *i*, *ii*, and *iii*, are possible. For the sake of simplicity we have chosen measures that are non-spatial giving rise to vertical efficiency frontiers. That is, for each measure there is a critical number of beneficiaries that has to be reached for the measure to be efficient. For measure *i* this number is *b*, for measure *ii* it is *d*, and for measure *iii* it is *f*. Planning measure *i* could be the introduction of bilingual names of public institutions in the jurisdictions under consideration, Measure *ii* could be a requirement to publish official information in the minority language in the relevant jurisdictions and measure *iii* the provision of a radio station broadcasting in the minority language in the jurisdiction considered. The efficiency frontiers of each of the three measures are shown in figure 4.3.

There are three possible one-measure categories with each of the three measures making up a separate category. That is, there is a possible policy consisting of three categories, each consisting of one single measure. There are two possible minimal-error categories with two measures, combining measures *i* and *ii*, or joining together measures *ii* and *iii*.²³ This implies two possible language policies, one with one category consisting of measures *i* and *ii* and the other category consisting of measure *iii* as well as one policy consisting of measure *i* in one category and measures *ii* and *iii* in the other one. Finally, there is one three-measure category making up a single-category policy. All in all, with the three planning measures we have four different relevant policy options. They are collected in table 4.1. The three measures cause benefits and costs in each jurisdiction where they are introduced. In this example the costs are fixed costs; we write them as c^i, c^{ii} , and c^{iii} . The average benefits *per capita* are determined by the planner and denoted by β^i, β^{ii} , and β^{iii} . (The *benefits* β^i, β^{ii} , and β^{iii} should not be confused with *category* β in the table and the following discussion.)

In this example the only sensible rule is to fix the minimum number of beneficiaries (n^*) necessary for the implementation of the corresponding category. For policy *I*, we would have a different criterion for each measure, since each measure is also a category: $n^{\alpha*} = b$, $n^{\beta*} = d$, and $n^{\gamma*} = f$. The result of the policy would be the application of category α (introduction of measure *i*) in all four jurisdictions; the application of category β (introduction of measure *ii*) in jurisdictions *B*, *C*, and *D*; and the application of category γ (introduction of measure *iii*) in jurisdiction *D*. There will be no planning errors of either type, but the administrative costs consists of administering three different categories that have to be checked against four

²³ The category consisting of measures *i* and *iii* is obviously more costly than the two others and therefore ignored.

Policy	Category	Planning measure	Administrative processes
<i>I</i>	α	<i>i</i>	12
	β	<i>ii</i>	
	γ	<i>iii</i>	
<i>II</i>	α	<i>i</i>	8
	δ	<i>ii, iii</i>	
<i>III</i>	ε	<i>i, ii</i>	8
	γ	<i>iii</i>	
<i>IV</i>	ζ	<i>i, ii, iii</i>	4

TABLE 4.1 Possible language policies with three planning measures

jurisdictions, in all 12 administrative processes.

For policy *II*, category α would have an optimal criterion $n^{\alpha*} = b$ as in policy *I*, giving rise to no error. For category δ we have to look for an optimal criterion (an optimal critical value for the size of the population in the jurisdictions) in the interval $d \leq n^{\delta*} \leq f$. There will then be no error with respect to jurisdictions *A*, *B*, and *D*; category δ would not be implemented in jurisdictions *A* and *B* and would be implemented in jurisdiction *D*, which in all three cases is the efficient outcome. For jurisdiction *C* there will be an inclusion error (E_i) due to measure *iii* if category δ is implemented in jurisdiction *C*, that is, if $n^{\delta*} \leq e$, and an exclusion error (E_e) due to measure *ii* if the category is not implemented in the jurisdiction, that is if the rule is strict enough, $n^{\delta*} > e$. These errors can easily be quantified and their size expressed in monetary units. We simply find the net benefits of the measure, $\beta^{ii}e - c^{ii} (< 0)$ or $\beta^{iii}e - c^{iii} (> 0)$, respectively. This is negative for measure *ii*; if measure *ii* is implemented, we implement a policy whose costs exceed the benefits and the net costs, that is the absolute value of the net benefits is the inclusion error. It is positive for measure *iii*; if measure *iii* is not introduced, we fail to realize potential benefits and the value of the net benefits is then the exclusion error:

$$d \leq n^{\delta*} \leq e: E_e = 0; \quad E_i = c^{iii} - \beta^{iii}e$$

$$e < n^{\delta*} \leq f: E_e = \beta^{ii}e - c^{ii}; \quad E_i = 0$$

Using the definition of the efficiency frontier, that is, net benefits are zero, $c^{ii} = \beta^{ii}d$ and $c^{iii} = \beta^{iii}f$, we can eliminate the costs from the expressions:

$$d \leq n^{\delta*} \leq e: E_e = 0; \quad E_i = \beta^{iii}(f - e)$$

$$e < n^{\delta*} \leq f: E_e = \beta^{ii}(e - d); \quad E_i = 0$$

The planner has to compare E_i and E_e , possibly giving different weights to the two types of error and then choose a decision criterion $n^{\delta*}$ that leads to the smallest error. This error, E^{II} , will then be compared to the administrative costs that now involve two categories that have to be checked for four jurisdictions, in all 8 administrative processes.

Looking at the errors in the form above, helps us understand the more general discussion in section 4.3. It is clear that the expressions give us the differences between costs and benefits for any choice of the critical number of beneficiaries necessary for the implementation or non-implementation of each measure of the category in the jurisdictions considered. The products $\beta^{ii}e$ and $\beta^{iii}e$ are the gross benefits of measures *ii* and *iii* if the measures are introduced in jurisdiction *C* (with the benefiting population of size *e*). The break-even sizes of the number of beneficiaries for the two measures are *d* and *f* (the efficiency frontiers). The corresponding costs are then *per definitionem* $c^i = \beta^{ii}d$ and $c^{ii} = \beta^{iii}f$, and the net loss by the non-implementation of measure *ii* in *C* – the exclusion error – is potential benefits minus costs: $\beta^{ii}(e - d)$. Similarly the net costs of the implementation of measure *iii* in *C* – the inclusion error – are costs minus benefits: $\beta^{iii}(f - e)$. In other words, the costs are given by the weighted distance of the jurisdiction from the efficiency frontiers in the $(\delta - n)$ diagram, and the weights are the importance of the measures expressed as *per capita* benefits.

Policy *III* is similar to policy *II*. The optimal criterion $n^{v*} = f$ is the same as in policy *I* and gives rise to no error. The errors for the different choices of $n^{\varepsilon*}$ are found analogously to $n^{\delta*}$ in policy *II*:

$$b \leq n^{\varepsilon*} \leq c: E_e = 0; \quad E_i = \beta^{ii}(d - c)$$

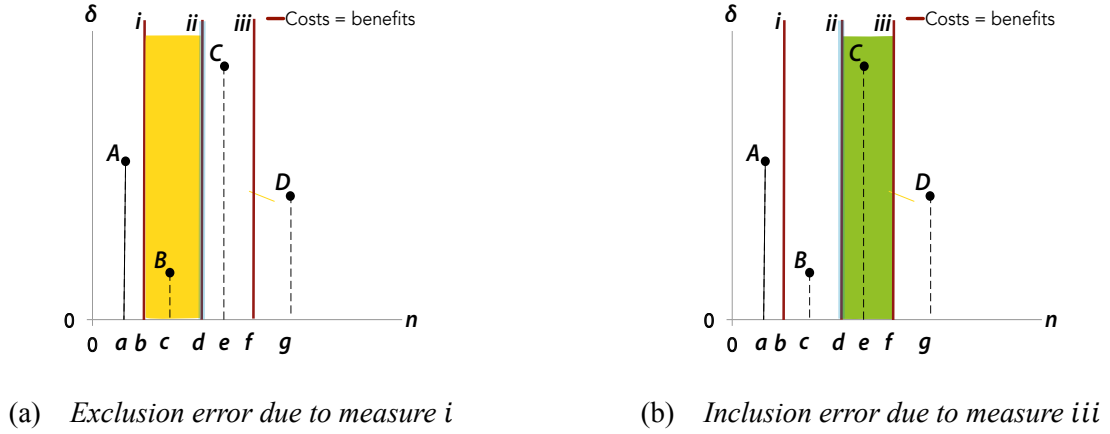
$$c < n^{\varepsilon*} \leq d: E_e = \beta^i(c - b); \quad E_i = 0$$

The planner again has to compare E_i and E_e in order to select the best $n^{\varepsilon*}$. The resulting error, E^{III} , will then be compared to the administrative costs that involve two categories that have to be checked for four jurisdictions, in all 8 administrative processes.

The comparison of policy *II* and *III* boils down to determining whether measure *ii* should join up with measure *i* or with measure *iii* to form a category. If the efficiency frontiers of measures *i* and *ii* are close; that is, *b* and *d* – and, hence, also *c* – are close, that speaks for joining measures *i* and *ii*, since $d - c$ and $c - b$ are then small and, as a consequence, also the error: policy *III* should be chosen. However, if the outlying measure *iii* is sufficiently unimportant, β^{iii} being small enough, the inclusion error in policy *II*, $\beta^{iii}(f - e)$, can in spite of the big difference between *e* and *f* become small enough that policy *II* will lead to the smaller error and be chosen as the preferred one. A preliminary conclusion is that closeness of cost structures, efficiency frontiers *b* and *d* being close, speaks for joining measures together, but only if the importance of the measures – as measured by the *per capita* benefits – is comparable to that of other measures.

The intuition is clear, If two important measures are joined together there will be implementation errors of a certain size caused by both measures. The closer are the efficiency frontiers, the smaller are these errors. If the measures are put in separate categories and combined with unimportant measures, the decision rule will be chosen on the basis of the important measures (close to their efficiency frontiers) leading to small implementation errors, which will be caused mainly by the unimportant measures. This error can then be sufficiently small to justify putting the important measures in separate categories. In deciding between the categories to which a measure should be added, there is – in other words – a trade-off between the closeness to the category in terms of the location of the efficiency frontiers in the $\delta - n$ -diagram and the importance of the measures already part of the category.²⁴ That is, in general one has to know both

²⁴ As an illustration choose $b = 2, c = 2.2, d = 3, e = 6$ and $f = 9$. Let all β be equal to 20. Policy *III* produces the minimal error. Now let $\beta^{iii} = 1$. In this case policy *II* has the lower costs.

FIGURE 4.4 Different errors for $n^{\zeta^*} = d$

the (attributed) individual benefit of the planning measures, the β , and the efficiency frontier of each planning measure, $n^i = c^i/\beta^i$ etc., in order to make a comparison of the costs of the two types of errors and consequently of the two policy alternatives.

Policy *IV* is somewhat more complicated. The policy choice is to introduce all three measures, category ζ , or none. For the policy criterion n^{ζ^*} we can distinguish five relevant ranges:

1. $0 < n^{\zeta^*} \leq a$: implementation of category ζ in A , B , C , and D
2. $a < n^{\zeta^*} \leq c$: implementation of category ζ in B , C , and D
3. $c < n^{\zeta^*} \leq e$: implementation of category ζ in C and D
4. $e < n^{\zeta^*} \leq g$: implementation of category ζ in D
5. $g < n^{\zeta^*}$: implementation of category ζ in no jurisdiction

The different choices give rise to different errors of the two types:

1. an inclusion error with respect to measure i in A , with respect to measure ii in A and B , and with respect to measure iii in A , B , and C
2. an inclusion error with respect to measure ii in B and with respect to measure iii in B and C
3. an exclusion error with respect to measure i in B ; an inclusion error with respect to measure iii in C
4. an exclusion error with respect to measure i in B and with respect to measures i and ii in C
5. an exclusion error with respect to measure i in B , with respect to measures i and ii in C , and with respect to measures i , ii , and iii in D

In figure 4.4, we illustrate the errors if the planner sets $n^{\zeta^*} = d$. Measure *i* will give rise to an exclusion error in jurisdiction *B*, and measure *iii* will give rise to an inclusion error in jurisdiction *C*.

The efficiency loss due to the errors can again be quantified. If $n^{\zeta^*} = d$, measure *iii* is introduced in jurisdiction *C* and the net benefits are $\beta^{iii}e - c^{iii}$. *Per definitionem* $\beta^{iii}f - c^{iii} = 0$. Hence, $\beta^{iii}e - c^{iii} < 0$, the net benefits are negative and the introduction of the measure causes an inclusion error of this size. Similarly, if measure *i* is not realized in jurisdiction *B*, we have an efficiency loss of $\beta^i c - c^i$ which is positive, since *per definitionem* $\beta^i b - c^i = 0$. The choice of $n^{\zeta^*} = d$ causes an exclusion error of this size.

The exclusion and inclusion errors for the different values of n^{ζ^*} are:

1. $E_e = 0$
 $E_i = [c^i - \beta^i a] + [2c^{ii} - \beta^{ii}(a + c)] + [3c^{iii} - \beta^{iii}(a + c + e)]$
2. $E_e = 0$
 $E_i = [c^{ii} - \beta^{ii}c] + [2c^{iii} - \beta^{iii}(c + e)]$
3. $E_e = \beta^i c - c^i$
 $E_i = c^{iii} - \beta^{iii}e$
4. $E_e = [\beta^i(c + e) - 2c^i] + [\beta^{ii}e - c^{ii}]$
 $E_i = 0$
5. $E_e = [\beta^i(c + e + g) - 3c^i] + [\beta^{ii}(e + g) - 2c^{ii}] + [\beta^{iii}g - c^{iii}]$
 $E_i = 0$

Using the definitions of the efficiency frontiers ($c^i = \beta^i b$ etc.), we can eliminate the expressions for the costs from the errors:

1. $E_e = 0$
 $E_i = \beta^i [b - a] + \beta^{ii} [2d - (a + c)] + \beta^{iii} [3f - (a + c + e)]$
2. $E_e = 0$
 $E_i = \beta^{ii} [d - c] + \beta^{iii} [2f - (c + e)]$
3. $E_e = \beta^i [c - b]$
 $E_i = \beta^{iii} [f - e]$
4. $E_e = \beta^i [c + e - 2b] + \beta^{ii} [e - d]$
 $E_i = 0$
5. $E_e = \beta^i [c + e + g - 3b] + \beta^{ii} [e + g - 2d] + \beta^{iii} [g - f]$
 $E_i = 0$

The various parameters are independent of one another. It is clear that the error in range 1 is greater than in range 2 and that in range 5 it is greater than in range 4. However, there is no clear ranking of ranges 2 through 4; each of them could be the one with the lowest aggregated error. The exclusion error increases, however, as n^* moves from range 2 to range 3, and as it moves from range 3 to range 4. That is, if the planner sufficiently strongly supports the minority

– and hence gives sufficiently low weight to the inclusion errors – the optimal policy would be to select $n^{\zeta*}$ in range 2: $a < n^{\zeta*} \leq c$. See section 4.4.

Like the cases of the other possible policies, the planner has to compare the various errors E_i and E_e in order to find the best $n^{\zeta*}$. The resulting error E^{IV} will then be compared to the administrative costs that now involve one category that have to be checked for four jurisdictions, in all 4 administrative processes.

4.2.1 Optimal criteria in general

Let the decision rule be of the type $r = (n, \delta)$; that is, the choice of policy is determined by the total size of a minority (n) and its density (δ) in the jurisdictions under consideration. The decision criterion then consists of two numbers giving the minima of the two parameters, $r^b = (n^b, \delta^b)$, that both have to be reached by a jurisdiction for the category to be enacted in it.

The size of the error due to a given measure i and attributable to a specific jurisdiction characterized by n and δ is by definition the product of $g^i(n, \delta)$ and β^i . β^i reflects the importance of planning measure i , and one can look upon $g^i(n, \delta)$ as an approximation of the distance in the $(\delta - n)$ -diagram between the jurisdiction and the efficiency frontier of measure i .²⁵ Looking for the total planning error of a category, we first have to find the error of each single measure summed over the relevant jurisdictions, and then we have to sum these aggregate errors over all measures in the category.

The total error due to a given measure is the sum of the errors of all affected jurisdictions, that is the jurisdictions between the efficiency frontier and the lines defined by (n^b, δ^b) , the shaded area in figure 4.1 (in the figure $(n^b, \delta^b) = (n^*, \delta^*)$). Since the β of any given measure by assumption is the same for all jurisdictions, the total error can be written as the product of β and the sum over over all relevant jurisdictions (characterized by their (n, δ)) of the corresponding $g(n, \delta)$. That is, for each planning measure the planning errors depend on the importance of the measure (β) and the sum of the (modified) distances of the efficiency frontier to the jurisdictions located in the area between the efficiency frontier and the lines defined by the decision criterion (n^b, δ^b) .

The total error of a category will then be the sum of these aggregated errors over all planning measures in the category. That is, it is a weighted sum of the aggregated (modified) distances of the jurisdictions to each efficiency frontiers. The aggregated errors are for each measure i weighted by the importance of the measure, β^i . The size of the total error of the category will then depend both on the closeness of the efficiency frontiers – the cost *structure*, represented by the different g -functions – and the importance of the different measures – represented by their β -values.

The optimal criterion for a given category can easily be characterized in general. Letting n^b or δ^b increase, the inclusion errors will decrease and the exclusion errors increase as the number of jurisdictions with an inclusion error is reduced and the number of jurisdictions with an exclusion error is increased. The optimal criterion will be reached when the decrease in the sum of inclusion errors of all measures in the category is exactly balanced by the increase in the sum of exclusion errors of the measures, possibly weighing the two types of error differently if

²⁵ If the efficiency frontier is described by a function $n^i(\delta)$, $c^i(n^i(\delta), \delta)/\beta = n^i(\delta)$, and $g^i(n, \delta) = n - n^i - [c(n, \delta) - c(n^i, \delta)]/\beta^i$. That is, g^i is the distance modified by the change in costs in dependence on n . If the costs are independent of n , g^i is the exact distance in the n -direction.

redistribution is desirable. In appendix E this discussion is made more stringent and the optimal criterion specified:

PROPOSITION 5 The optimal criterion $r^{b*} = (n^{b*}, \delta^{b*})$ of a decision rule of type $r = (n, \delta)$ for a category b is given as the solution of equations equalizing, in the case of n , on the one hand, n^b with the weighted sums of the $g^i(n^b, \delta)$ over the planning measures i in category b and, in the case of δ , on the other hand, equalizing δ^b with the weighted sum of the $g^i(n, \delta^b)$ over the planning measures i in category b , the sums being averaged over the distribution of δ among the jurisdictions with the minority population (approximately) equal to n^{b*} and of n among the jurisdictions with the minority population density (approximately) equal to δ^{b*} , respectively. The weights of the sums are given by the importance of the measures represented by the *per capita* benefits β .

In appendix E it is also shown that in the case of non-rival and non-spatial costs (like in our simple example) the optimal criterion becomes very simple and can be written as a weighted sum of the values of n on the efficiency frontiers, n^i :

$$n^{b*} = \sum_{i \in I^b} \gamma^i n^i \quad (4.1)$$

The weights are given by the importance of the measures:

$$\gamma^i = \frac{\beta^i}{\sum_{i \in I^b} \beta^i} \quad (4.2)$$

COROLLARY 5A The efficient decision criterion for a category when costs are non-rival and non-spatial requires that n^{b*} be equal to the weighted average of the critical values of n of the different planning measures in the category with the weights given by the benefits of the measures.

4.2.2 Quantification of the errors of a category

Intuitively it is clear that the size of the errors of a given planning measure will be determined by the size of the areas between the efficiency frontier and the lines defined by the decision criterion as well as by the number of jurisdictions and the size of the benefits of the measures. Assume that the area of the errors increases, doubling the distance between the efficiency frontier and a fixed decision criterion n^* . The distance of the average relevant jurisdiction to the efficiency frontier will then double if the jurisdictions are evenly distributed in the $(\delta - n)$ -diagram. By constant costs, the planning error will be proportional to the distance in the diagram between the jurisdiction and the efficiency frontier. Hence, the error of the average jurisdiction is doubled. At the same time, the number of jurisdictions affected is also doubled. The total error is as a consequence increased by a factor of four. At the same time, the error is proportional also to the importance of the issue as reflected in the value of β .

As a consequence of the greater area, we can, hence, expect the errors to increase approximately with the square of the size of the distance between the efficiency frontiers and the lines fixed by the decision criterion as well as being proportional to the corresponding β , the *per capita* size of the benefit of the measure. For the aggregate errors in a category the individual errors have to be added. We should hence expect the category error to depend approximately on the sums of the benefits of the measures weighted by the squares of the average distance of the line representing the decision criterion to the efficiency frontier.

In appendix F a lower limit of the optimal error with decision rule $r = (n, \delta)$ in a given category is found. It can be written as the product of the aggregated importance of the measures in the category and a generalized weighted variance. This variance is essentially given as a weighted average of the squares of average distances (adjusted for costs) between the efficiency frontiers and the planning criterion of the measure. It can be seen as an indicator of the closeness of the cost structures of the measures in the category. That is, how close the efficiency frontiers are to one another. That is, there is a trade-off between the importance of the measures evaluated as the size of the benefits β^i and the closeness of the measures. To a first approximation, a single measure's contribution to the total error of the category it belongs to depends both on its β and on the distance between its efficiency frontier and the average frontier of the category. We have:

PROPOSITION 6 A certain measure's contribution to the size of the error in its category depends both on the closeness of the cost structure of the measure to the average in the category and on the size of the benefits of the measure.

In the case of non-rival and non-spatial cost structures the *exact* expression for the size of the planning errors can easily be found and is very simple:

$$E^b = \frac{1}{2} \sum_{i \in I^b} \beta^i [n^{b*} - n^i]^2 m \quad (4.3)$$

The parameter m is here the density of jurisdictions in the n -dimension of the $(\delta - n)$ -diagram. That is, it is the number of jurisdictions with a minority population size between n_1 and n_2 for any n_1 and n_2 with the property $n_2 - n_1 = 1$. The trade-off between the importance of a measure expressed by β^i and the distance between the efficiency frontier and the average efficiency frontier represented by the decision criterion n^{b*} becomes very clear:

COROLLARY 6A In order to minimize planning errors in the case of non-rival and non-spatial cost structures one should distribute important measures on the various categories such that their critical values of n (their efficiency frontiers) in each category are close. Unimportant measures can then be assigned residually.

4.3 SOME GENERAL PROPERTIES OF CATEGORIES SUBJECTED TO AN OPTIMAL POLICY

We first show that the planning costs decrease if a category with an optimal planning rule is divided into two new categories, each with an corresponding optimal rule. We then discuss how measures should be allocated to different categories.

By dividing a category into two parts, we introduce more flexibility into the system, since we now can have different policy rules for each of the different parts. By applying the rule of

the original category to the new ones, the planning costs would not change due to the splitting of the original category. By introducing separate optimal rules for the new categories, we can only lower the costs (unless by chance the new optimal rules are identical to the old one). In other words, with the big category we would have only one rule which would have to be a compromise of the rules in the two separate categories. Hence, we can reach a lower level of planning costs with the new categories in comparison to the original one. Note that the result is independent of the specific planning rule applied. By choosing measures that are close in the sense of proposition 6 within each of the two new categories and further apart from measures in the other category, the costs will certainly decrease. In appendix G this is formally shown. We can formulate:

PROPOSITION 7 The total costs of the planning errors decrease if a category to which an optimal decision rule is applied is divided into two categories, each with its own optimal decision rule. The result is independent of the kind of rule used and of planner objective.

The implications of this proposition can easily be confirmed in our small example where policy *IV* has higher optimal planning errors than policy *II* or *III*, and those in turn have higher optimal planning errors than policy *I*. See also table 4.3 below.

We have to address the issue, which measures should be assigned to which category when the number of categories is given. We have seen that the increase in the planning error of a category when a new measure is added to it depends on the importance of the new measure, on the number of jurisdictions affected, and on the distance of its efficiency frontier to the average of the other efficiency frontiers adjusted for costs. Making the *Gedankenexperiment* that we remove a planning measure from one category and add it to another, we would like to find out if the sum of the planning errors of the two categories increases or decreases – or, in other words, which measures should be moved from which category in order to decrease the sum of errors. Intuitively, if a measure is an “out-layer” in one category and more “central” in another it should be moved to the category where it is central.

There are two effects to consider when a measure is removed from a category. The error of the category is reduced by the error directly attributable to the measure and in addition the optimal rule is affected, decreasing the errors caused by the remaining measures, see proposition 7. By the same token, adding a measure to a category will increase the error of the category with the error directly attributable to the measure being added under the prevailing optimal rule, but this optimal rule will be adjusted, making the increase in the total error of the category smaller than the error attributable to the new measure under the prevailing rule. The size of the change in the error due to the modified rule will depend on the importance of the measures already in the category compared to the importance of the new measure.

This allows us to develop a test for whether a measure should be transferred from a category b_1 to a category b_2 . If we remove measure i from b_1 , the decrease of the error in category b_1 under the new optimal rule will be at least as big as the size of the error due to measure i under the old optimal rule. This conclusion holds since in addition to the direct reduction in the error in b_1 due to the removal of measure i , there is also the adjustment of the optimal rule for the new

category b'_1 without measure i .²⁶ By the same token, adding measure i to b_2 causes an increase in the error of category b_2 due to the addition of measure i and a change in the optimal rule, since the composition of the category is changed. This can be analyzed as a direct contribution equal to the error of measure i under the old rule of category b_2 followed by a change in the optimal rule of the category including measure i . The change in the optimal rule will, taken alone, reduce the total error of the category. Hence, the total error of category b_2 will increase by less than the error of measure i under the original rule of category b_2 .²⁷ Moving planning measure i from b_1 to b_2 will hence lower the total size of the errors of the language policy if the error due to measure i under the original rule of category b_1 is larger than under the original rule of category b_2 .²⁸ This gives us an amendment to proposition 6:

PROPOSITION 8 If there exist two categories b_1 and b_2 with corresponding optimal decision criteria, then there should be no individual measure in b_1 that would give rise to a lower error evaluated by the optimal criterion of b_2 than by the optimal criterion of b_1 . Inversely, there should be no individual measure in b_2 giving rise to a lower error under the optimal criterion in b_1 than under the optimal criterion of b_2 . The result holds independently of the kind of decision rule being used.

This gives us a necessary but not sufficient condition for the allocation of planning measures to different categories. That is, in general there will be measures left in the categories after this condition is fulfilled which should be reallocated in order to minimize the total size of planning errors.

By grouping measures that are similar in the structure of costs, the errors can be reduced. An optimal allocation of planning measures to categories should attempt to minimize distance between the efficiency frontiers in the $(\delta - n)$ -diagram, weighing the distances with the importance of the measures. Proposition 8 helps us relate the distances to the errors of the single measures. That is, it gives a marginal (local) condition for allocating measures to categories. However, it is not exhaustive due to the fact that the optimal rules change as the composition of categories is changed, and the change in the optimal rules is related to the importance of the measures.

²⁶ Let the size of the error due to measure i under the old optimal rule be $E^i(r^{b_1*})$ and let the decrease in the error of category b_1 due to the removal of measure i and the adjustment of the optimal rule be $-\Delta E^{b_1-*}$. Then we must have:

$$-\Delta E^{b_1-*} \geq E^i(r^{b_1*}) \quad (4.4)$$

²⁷ Let the error of measure i under the original rule of category b_2 be $E^i(r^{b_2*})$ and let the increase in the error of the category due to the new measure and the adjustment of the rule be ΔE^{b_2+*} . Then, we have:

$$E^i(r^{b_2*}) \geq \Delta E^{b_2+*} \quad (4.5)$$

$$(E^i(r^{b_1*}) \geq E^i(r^{b_2*})) \Rightarrow (-\Delta E^{b_1-*} \geq \Delta E^{b_2+*}) \quad (4.6)$$

4.3.1 An illustration

An illustration with the help of our small example can throw some additional light on propositions 6 and 8, as well as on their limitations. Consider the choice between policies *II* and *III*. The question here is to which category measure *ii* should be assigned. In policy *II*, it is joined to measure *iii* to form category δ ; in policy *III*, it forms category ε together with measure *i*. Or one could say that in policy *II* it is added to category γ which thereby converts to category δ , and in policy *III* it is added to category α which then becomes category ε . When the measure is added or removed from a category, the errors in the category will increase or decrease, respectively, under given decision criteria. In addition, however, the decision criterion might change, leading to a reduction of the errors in the new categories.

The optimal decision criteria for the different categories can easily be found. In determining the optimal criterion in category δ , the planner has to choose between an inclusion error in jurisdiction C due to measure *iii* when the decision criterion is less than e , and an exclusion error in jurisdiction C due to measure *ii* for a decision criterion greater than e . The errors will be $\beta^{iii}(f - e)$ and $\beta^{ii}(e - d)$, respectively. The relative size of the two errors will depend on the importance of the measures (reflected in the size of the *per capita* benefits β) and the distance between the efficiency frontiers of the measures and the jurisdictions in the $(\delta - n)$ -diagram ($f - e$ and $e - d$). Assuming that the planner gives equal weight to inclusion and exclusion errors, the decision criterion will be found by a simple comparison of the two errors. If measure *ii* is important enough – or its efficiency frontier lies far enough away from the size of jurisdiction C –, the planner will choose the inclusion error and a decision criterion smaller than e , for instance d . The choice of criterion is determined such that the contribution of measure *ii* to the planning error in the category will be zero; that also means that if measure *ii* is removed from the category the decision criterion will be changed to f , removing the inclusion error.

In the opposite case, the decision criterion is greater than e , say f , and the planning error due to measure *ii* is the exclusion error $\beta^{ii}(e - d)$. In this case, the removal of measure *ii* from the category will lower the category error by $\beta^{ii}(e - d)$ but have no influence on the decision criterion in the reduced category.

Mutatis mutandis the same holds for category ε . The optimal criteria of categories α and γ are given by b and f , respectively. The possible relative sizes of the two errors in the different policies, the optimal criteria, the category errors (E), and the contribution of measure *ii* to the category errors (E^{ii}) are listed in table 4.2.

Applying criterion b – the optimal criterion of category α – to measure *ii*, the measure would be implemented in jurisdiction B , causing an inclusion error equal to $\beta^{ii}(d - c)$. Applying criterion f – the optimal criterion of category δ – to measure *ii*, it would not be implemented in jurisdiction C , causing an exclusion error equal to $\beta^{ii}(e - d)$. These are the direct errors measure *ii* would cause if we change from policy *II* to *III* or *vice versa*. In table 4.2, these errors are shown in column E_{alt}^{ii} . In addition, if there is a change in the optimal decision criterion, column Criterion change, this will also alter the errors after a change, increasing the reduction in the category of departure and reducing the increase in the category of arrival. Given the different possibilities in the relations between the potential exclusion and inclusion errors in the two categories, we can distinguish four different cases.

Case *C* in the table is the simplest one and the one where we always become a clear result. Here measure *ii* is relatively unimportant in both possible categories. It influences the optimal decision criterion in neither of the two categories it can belong to. Only the direct contribution

Case	Error relation	Criterion	E	E^{ii}	E_{alt}^{ii}	Criterion change	ii near B C
A	$II: \beta^{ii}(e-d) > \beta^{iii}(f-e)$	$n^{\delta*} = d$	$\beta^{iii}(f-e)$	0	$\beta^{ii}(d-c)$	yes	? II
	$III: \beta^i(c-b) > \beta^{ii}(d-c)$	$n^{\varepsilon*} = b$	$\beta^{ii}(d-c)$	$\beta^{ii}(d-c)$	$\beta^{ii}(e-d)$	no	
B	$II: \beta^{ii}(e-d) > \beta^{iii}(f-e)$	$n^{\delta*} = d$	$\beta^{iii}(f-e)$	0	$\beta^{ii}(d-c)$	yes	? ?
	$III: \beta^i(c-b) < \beta^{ii}(d-c)$	$n^{\varepsilon*} = d$	$\beta^i(c-b)$	0	$\beta^{ii}(e-d)$	yes	
C	$II: \beta^{ii}(e-d) < \beta^{iii}(f-e)$	$n^{\delta*} = f$	$\beta^{ii}(e-d)$	$\beta^{ii}(e-d)$	$\beta^{ii}(d-c)$	no	III II
	$III: \beta^i(c-b) > \beta^{ii}(d-c)$	$n^{\varepsilon*} = b$	$\beta^{ii}(d-c)$	$\beta^{ii}(d-c)$	$\beta^{ii}(e-d)$	no	
D	$II: \beta^{ii}(e-d) < \beta^{iii}(f-e)$	$n^{\delta*} = f$	$\beta^{ii}(e-d)$	$\beta^{ii}(e-d)$	$\beta^{ii}(d-c)$	no	III ?
	$III: \beta^i(c-b) < \beta^{ii}(d-c)$	$n^{\varepsilon*} = d$	$\beta^i(c-b)$	0	$\beta^{ii}(e-d)$	yes	

TABLE 4.2 Choice between policy II and III

to the implementation error matters. If it belongs to category δ it is $\beta^{ii}(e-d)$ and if it belongs to category ε it is $\beta^{ii}(d-c)$. The relative distance from its efficiency frontier to the jurisdictions affected determines in which category measure ii causes the smallest error and consequently which policy should be chosen. If the frontier is closer to jurisdiction B , $e-d < d-c$ and policy III should be chosen. In the reverse case policy II should be elected.

Generally speaking, and according to proposition 8 if $E^{ii} > E_{alt}^{ii}$, measure ii should be removed from the present category and put in the other, since the reduction in the error in the first category will be greater than or equal to E^{ii} and the increase in the error of the second category will be less than or equal to E_{alt}^{ii} . This can be applied to case A if the efficiency frontier of measure ii is closer to jurisdiction C than to B ($d-c > e-d$), and to case D if the efficiency frontier of measure ii is closer to jurisdiction B than to C ($e-d > d-c$). In all other cases, information only about measure ii and the jurisdictions is not enough to find a result. One needs to know somethings about the two other measures in addition.

4.3.2 A basic trade-off

In appendix H, the distribution of measures on categories is investigated in more detail. It is found that there is a trade-off between the similarity of the cost structure of the individual measure and that of a potential category for the measure on the one hand and the relative importance of the possible categories on the other hand. Then cost structure is mirrored in the decision criterion and the importance in the *per capita* benefit. We can state:

PROPOSITION 9 Let there be two categories b_1 and b_2 with different optimal decision criteria r^{b_1*} and r^{b_2*} as well as a measure k with optimal decision criterion r^{k*} .

1. If $r^{k*} = r^{b_1*}$ then, in order to minimize the size of the planning error, measure k should be added to category b_1 .
2. If $r^{k*} \neq r^{b_1*}$ then there exists a number $B > 0$, such that if $\sum_{i \in I_{b_2}} \beta^i \leq B$ measure k should be added to category b_2 in order to minimize the size of the planning error.

Due to the continuity of the cost functions we can extend proposition 9:

COROLLARY 9A For given values of the various *per capita* benefits, if the cost structures (as mirrored in the optimal planning criteria) of the measure k and one of the categories are sufficiently close it is optimal to add measure k to the category in question.

Loosely speaking, the different categories should contain measures of similar cost structures.

COROLLARY 9B For given and different cost structures (as mirrored in the optimal planning criteria) of the measure k and the categories, if the importance of the categories (as mirrored in the sum of the benefits) is sufficiently different it is optimal to add measure k to the category with the lowest importance.

The somewhat imprecise implication is that the different categories should be of similar importance.

4.4 CHOICE OF POLICY

In order to choose the best policy (in his own eyes), the planner first has to decide on the redistribution issue in order to calculate the size of the relevant total error of each possible policy. The exclusion errors are to the disadvantage of the minority population; efficient planning measures to the advantage of the minority population are rejected. The inclusion errors, however, benefit members of the minority; inefficient measures to their advantage are approved of. By weighing the two types of error against one another, the planner can conduct a policy that is more or less friendly to the minority population. If members of a minority have less rights than members of the majority this can be seen as a social cost that has to be added to the costs of the two errors before searching the minimum errors of a policy and *vice versa* if a certain discrimination of minorities is considered a social benefit in the eyes of the planner.

Two extreme positions are to judge the errors lexicographically. An extremely minority adverse (but still rational) planner would first select the policy rule for a given policy that minimize the inclusion error and then among the remaining possibilities choose the one that minimizes the exclusion error. By the same token, an extremely minority friendly planner would first minimize the exclusion error and then among the remaining possibilities find the one that leads to the smallest inclusion error.

As an example, consider policy *IV* in section 4.2 above. The minority adverse planner would in the first round choose the policy rule in ranges 4 and 5, which lead to an inclusion error equal to zero. He would then choose the policy rule in range 4, thereby minimizing the exclusion error in the two ranges. An extremely minority friendly planner would instead in the first round minimize the exclusion error and choose ranges 1 and 2 for the policy rule making the exclusion error zero and then choose the policy rule in the range with the lowest inclusion error, namely in range 2.

In general, as is shown in appendix D.3, the choice problem can be included in the cost-benefit analysis by assigning weights to the different types of error. In the case of a distributionally neutral planner, she would simply try to minimize the sum of the errors of the two types: $E_e + E_i$. In the cases of an extremely minority-adverse planner or an extremely minority-friendly planner, the lexicographical principle would be used.

For the sake of simplicity we assume in our example that the planner is extremely minority friendly and sees exclusion errors as a much bigger problem than inclusion errors. That is, he strongly favors giving too many rights to a minority over giving it too few rights. In our example we bring this to the extreme in assuming that the planner has lexicographic preferences in the evaluation of the two types of error.

In finding an optimal critical value for the policy rule for a given category, he will according to the lexicographical preferences first minimize the exclusion error and thereafter choose the rule with the lowest inclusion errors. Assuming a lexicographical planner objective, we can quantify the errors under the optimal rule for each policy. This has been done in table 4.3. In the table we have also listed the optimal critical values for the rule for each category and the number of administrative processes needed in order to implement each policy. It is readily seen that the size of the errors decreases with the number of categories of the policy. On the other hand, the number of administrative processes increases with the number of categories.

In choosing an optimal policy, the planner will have to compare the size of the weighted errors with the costs of administering the policy. An optimal policy is then a policy for which the sum of these costs is minimized.

In our example, we have simplified the problem of choosing an optimal policy drastically for the purpose of providing a simple exposé of the main ideas behind a cost-based optimal language policy. A more general structure of the problem of finding an efficient language policy is outlined in appendix D. However, this general treatment does not change the qualitative aspect of the problem in any essential way. The basic ideas are all present in our small (and admittedly trivial) illustrative example.

In this example we have focused on the size of the costs, assuming the structure to be the same, namely non-rival non-spatial. This leads to an optimal composition of the policy with a certain number of categories, each category characterized by a similar size of the errors of the planning measures contained in it when evaluated in terms of the size of the respective benefits. See corollary 6a. This way, costs due to errors of the two kinds are minimized. In general, however, we also have to consider the structure of the costs. Proposition 6 shows that in an optimal policy we will in each category have planning measures that are close both in structure and costs measured in units of the benefits.

Pol- icy	Optimal rule	Error size	Number of administrative processes
I	$n^{\alpha*} = b$ $n^{\beta*} = d$ $n^{\gamma*} = f$	$E_e = 0$ $E_i = 0$	12
II	$n^{\alpha*} = b$ $d \leq n^{\delta*} \leq e$	$E_e = 0$ $E_i = \beta^{iii}(f - e)$	8
III	$b \leq n^{\varepsilon*} \leq c$ $n^{\gamma*} = f$	$E_e = 0$ $E_i = \beta^{ii}(d - c)$	8
IV	$a < n^{\zeta*} \leq c$	$E_e = 0$ $E_i = \beta^{ii}[d - c] + \beta^{iii}[2f - (c + e)]$	4

TABLE 4.3 Optimal rules, error sizes and administrative costs of language policies

5 CONCLUSIONS

By focusing on the costs of language planning measures we have seen that two parameters become important for a rational language policy, the number of beneficiaries of the policy and their geographical distribution in the jurisdictions under consideration. The commonly used percentage rule has no theoretical basis. We also argue that due to different cost structures and cost magnitudes of different planning measures, an optimal language policy will consist of several different categories of planning measures, each category having its own policy rule. We have also outlined, how this optimal categorization can be found.

Methodologically there are several levels of trade-offs between costs and benefits. On the level of planning measures there is a trade-off between simple applicable rules and efficiency. By restricting the policy tools to the application of a few indicators administrative ease is won at the cost of inefficiency. This inefficiency consists of two types of planning errors: efficient planning measures are sometimes not realized and some inefficient measures are introduced. The second trade-off is the choice of the planner between these two types of inefficiencies. This choice is closely related to distributional issues between the minority and majority populations. The non-realization of efficient measures are to the disadvantage of the linguistic minority, whereas the introduction of inefficient measures is to its benefit. The third trade-off is between having a differentiated policy with many categories and the administrative costs of administering several policy categories. Having several measures, each having similar cost structures and magnitudes, introduces more flexibility and as a consequence leads to lower aggregate planning errors.

6 REFERENCES

BUCHANAN, JAMES M. (1987). "The constitution of economic policy". In: *American Economic Review* 77: 243–250.

- BUCHANAN, JAMES M. and GORDON TULLOCK (1962). *The calculus of consent*. Ann Arbor: The University of Michigan Press.
- DESGAGNÉ, MAXIME LEBLANC and FRANÇOIS VAILLANCOURT (2016). “The distribution of the benefits and costs of Canada’s federal official languages policy: Results for 2001”. In: *The economics of language policy*. Edited by MICHELE GAZZOLA and BENGT-ARNE WICKSTRÖM. Cambridge: MIT Press. Chapter 4: 141–164.
- GAZZOLA, MICHELE (2014). *The evaluation of language regimes. Theory and application to multilingual patent organisations*. Amsterdam: John Benjamins.
- GAZZOLA, MICHELE and FRANÇOIS GRIN (2017). “Comparative language policy and evaluation: Criteria, indicators and implications for translation policy”. In: *Translation and public policy: Interdisciplinary perspectives and case studies*. Edited by GABRIEL GONZÁLEZ NÚÑEZ and REINE MEYLAERTS. Routledge advances in translation and interpretation 23. New York: Routledge. Chapter 5: 83–116.
- GINSBURGH, VICTOR A. (2017). “Contingent valuation, willingness to pay, and willingness to accept”. In: *Economic ideas you should forget*. Edited by BRUNO S. FREY and DAVID ISELIN. Cham: Springer International Publishing: 65–66.
- GRIN, FRANÇOIS (2003). “Language planning and economics”. In: *Current Issues in Language Planning* 4.1: 1–66.
- MOREY, EDWARD R. (1984). “Confuser surplus”. In: *American Economic Review* 74.1: 163–173.
- MUSGRAVE, RICHARD A. (1956/1957). “A multiple theory of budget determination”. In: *Finanzarchiv* 17.3: 333–343.
- NG, YEW-KWANG (2004). *Welfare economics: Towards a more complete analysis*. Houndmills: Palgrave Macmillan.
- SCITOVSKY, TIBOR DE (1941). “A note on welfare propositions in economics”. In: *Review of Economic Studies* 9.1: 77–88.
- VAILLANCOURT, FRANÇOIS (1997). “Economic costs and benefits of the official languages: Some observations”. In: *Official languages and the economy: Papers presented at a colloquium hosted by the Official languages support programs branch, Department of Canadian Heritage, held on May 5, 1995*. New Canadian perspectives. Ottawa: Canadian Heritage: 103–118.
- (2018). “Linguistic justice in a federal setting: The case of Canadian provinces”. In: *Language policy and linguistic justice: Economic, philosophical and sociolinguistic approaches*. Edited by MICHELE GAZZOLA, TORSTEN TEMPLIN, and BENGT-ARNE WICKSTRÖM. Heidelberg: Springer. Chapter 7. Forthcoming.

WICKSTRÖM, BENGT-ARNE (2016). “Language rights: A welfare-economics approach”. In: *The Palgrave handbook of economics and language*. Edited by VICTOR A. GINSBURGH and SHLOMO WEBER. Houndmills: Palgrave Macmillan. Chapter 22: 659–688.

WICKSTRÖM, BENGT-ARNE, MICHELE GAZZOLA, and TORSTEN TEMPLIN (2018a). “Can the ‘free market’ manage language diversity?” In: *The MIME vademecum: Mobility and inclusion in multilingual Europe*. Edited by FRANÇOIS GRIN, MANUEL CÉLIO CONCEIÇÃO, PETER A. KRAUS, LÁSZLÓ MARÁCZ, ŽANETA OZOLIŅA, NIKE K. POKORN, and ANTHONY PYM. Geneva: MIME Project. Chapter 3: 34–35.

— (2018b). “Do costs matter in language policy?” In: *The MIME vademecum: Mobility and inclusion in multilingual Europe*. Edited by FRANÇOIS GRIN, MANUEL CÉLIO CONCEIÇÃO, PETER A. KRAUS, LÁSZLÓ MARÁCZ, ŽANETA OZOLIŅA, NIKE K. POKORN, and ANTHONY PYM. Geneva: MIME Project. Chapter 11: 50–51.

WICKSTRÖM, BENGT-ARNE, TORSTEN TEMPLIN, and MICHELE GAZZOLA (2018). “An economics approach to language policy and linguistic justice”. In: *Language policy and linguistic justice: Economic, philosophical and sociolinguistic approaches*. Edited by MICHELE GAZZOLA, TORSTEN TEMPLIN, and BENGT-ARNE WICKSTRÖM. Heidelberg: Springer. Chapter 1: 3–64.

APPENDICES

A A POSSIBLE SPECIFICATION OF THE COST FUNCTION

We write the costs of a certain policy measure providing language rights of a certain quality to all beneficiaries as a (sufficiently differentiable) function $\chi(n, a; \rho, \sigma)$, defined for $n \geq 0$ and $a \geq 0$, with the properties:^{29 30}

$$\begin{aligned}\chi(0, 0; \rho, \sigma) &= \theta \geq 0 \\ \chi_n(n, a; \rho, \sigma) &\geq 0, \chi_a(n, a; \rho, \sigma) \geq 0 \\ \chi_{nn}(n, a; \rho, \sigma) &\leq 0, \chi_{aa}(n, a; \rho, \sigma) \leq 0, \chi_{na}(n, a; \rho, \sigma) = 0\end{aligned}\tag{A.1}$$

That is, we assume that there are some fix costs, $\theta \geq 0$, of a planning measure combined with concave variable costs that are additively separable in the two arguments. The parameters ρ and σ were introduced in table 3.1 and are indicators of the strength of the dependence of χ on the two variables n and a respectively. Specifically, we write:

$$\begin{aligned}\chi(n, a; 0, 0) &= \theta \geq 0 \\ \chi_n(n, a; 0, \sigma) &= 0, \chi_a(n, a; \rho, 0) = 0 \\ \chi_n(n, a; 1, \sigma) &= \nu > 0, \chi_a(n, a; \rho, 1) = \alpha > 0\end{aligned}\tag{A.2}$$

²⁹ χ is clearly sub-additive, since for different territories and number of beneficiaries, the planning measure can be introduced separately on the two territories at the sum of the individual costs. If there are any synergies the total cost will be reduced. Let $x := (n, a)$, $x_1, x_2 \geq 0$. We have $\chi(x_1 + x_2; \rho, \sigma) \leq \chi(x_1; \rho, \sigma) + \chi(x_2; \rho, \sigma)$. It is reasonable to assume the costs to be concave in the two variables. This is achieved if the costs are given by the sum of two concave functions in n and a , respectively, as well as a term representing the fixed costs.

³⁰ We use subscripts to denote partial derivatives if there is no risk of confusion.

That is, θ are the fixed costs, ν the (constant) marginal costs of an additional beneficiary in the case of proportionality in n ($\rho = 1$), and α the (constant) marginal costs of an increase in the area of application of the policy in the case of proportionality in a ($\sigma = 1$). We will in general suppress the dependence on the parameters ρ and σ for the sake of keeping the notation transparent.

Let β be the average propensity to pay for the planning measure, or – for our argument equivalently – the planner's evaluation of the *per capita* benefit of the planning measure. Then the planning measure has a non-negative benefit-cost difference if $\beta n - \chi(n, a) \geq 0$. Normalizing, we define $f(n, a)$ by:

$$f(n, a) := n - \frac{\chi(n, a)}{\beta} \quad (\text{A.3})$$

We can also express the cost parameters in normalized units defining $\kappa := \nu/\beta$ as the marginal costs of an additional beneficiary in the case of $\rho = 1$, $\lambda := \alpha/\beta$ gives us the corresponding marginal costs of an increase in the area when $\sigma = 1$, and $\mu := \theta/\beta$ the fixed costs of the measure, all expressed in units of the average evaluation of the policy measure. In the illustrations the values of these parameters are listed in addition to those of ρ and σ . The efficiency condition is now $f(n, a) \geq 0$. We are interested in the restrictions this condition imposes on n and a . It turns out that the restrictions on n and a implied by efficiency can be rather complex. Much easier restrictions are obtained if we substitute n/δ for a , $\delta := n/a$ being the population density of the minority-language speakers. We define:

$$c(n, \delta) := \chi(n, n/\delta) \quad (\text{A.4})$$

and finally write

$$g(n, \delta) := f(n, n/\delta) = n - \frac{c(n, \delta)}{\beta} \quad (\text{A.5})$$

Note that $c_n \geq 0$ and $c_\delta \leq 0$.

In order to simulate intermediate cases (and to define ρ and σ for intermediate values), we specify the cost function as:

$$\chi(n, a) := \nu [(1 - \rho + n)^\rho - (1 - \rho)^\rho] + \alpha [(1 - \sigma + a)^\sigma - (1 - \sigma)^\sigma] + \theta \quad (\text{A.6})$$

The function $g(n, \delta)$ then becomes:

$$g(n, \delta) = n - \kappa [(1 - \rho + n)^\rho - (1 - \rho)^\rho] - \lambda \left[\left(1 - \sigma + \frac{n}{\delta}\right)^\sigma - (1 - \sigma)^\sigma \right] - \mu \quad (\text{A.7})$$

This cost function seems to be the analytically simplest one that is additively separable in concave functions of n and a as well as in the fixed costs. It is also readily seen that for the corresponding choices of the parameters ρ and σ the special cases analyzed for the parameter values zero and one are obtained. The parameters ρ and σ are closely related to the concavity of the dependency of the costs on n and a .

The lower bounds of the values of n and δ on the efficiency frontier are readily calculated for our specification of the cost function. These values are given in table A.1.

For the table, we have defined \tilde{n}^{**} ($= n^{**}$ by construction) as the solution to equation:

$$n - \kappa [(1 - \rho + n)^\rho - (1 - \rho)^\rho] - \mu = 0 \quad (\text{A.8})$$

This solution exists if $\kappa < 1$.

(ρ, σ)	n^{**}	δ^{**}
$(0, \sigma), \sigma \neq 1$	μ	0
$(0, 1)$	μ	λ
$(\rho, \sigma), \sigma \neq 1$	\tilde{n}^{**}	0
$(\rho, 1), \rho \neq 1$	\tilde{n}^{**}	λ
$(1, \sigma), \sigma \neq 1$	$\mu/(1 - \kappa)$	0
$(1, 1)$	$\mu/(1 - \kappa)$	$\lambda/(1 - \kappa)$

TABLE A.1 Lower bounds of the values of n and δ on the efficiency frontier

B PROOF THAT THE EFFICIENCY FRONTIER IS A NON-POSITIVELY SLOPED CURVE EVERYWHERE

The slope s of the efficiency frontier is given by the negative of the ratio of the partial derivatives of g :

$$s := \left. \frac{d\delta}{dn} \right|_{g=0} = - \frac{\frac{\partial g}{\partial n}}{\frac{\partial g}{\partial \delta}} = - \frac{1 - \frac{\chi_n}{\beta} - \frac{\chi_a}{\beta\delta}}{\frac{n}{\chi_a \delta^2}} \quad (\text{B.1})$$

If $\chi_a = 0$ for $g(\tilde{n}, \tilde{\delta}) = 0$ the slope is infinite and the frontier vertical at this point. For $\chi_a > 0$ the denominator is clearly positive for positive values of n . We have to analyze the nominator. Taking the second derivative g_{nn} we find:

$$g_{nn} = -\frac{\chi_{nn}}{\beta} - \frac{\chi_{aa}}{\beta\delta^2} \geq 0 \quad (\text{B.2})$$

Assume that $g(0, \delta) < 0$. On the efficiency frontier $g(n, \delta) = 0$. Hence, if for a given $\delta = \tilde{\delta}$, $g(n, \tilde{\delta}) = 0$ has a smallest solution at $\tilde{n} \geq 0$, g approaches this solution from below and the derivative g_n at $(\tilde{n}, \tilde{\delta})$ must be non-negative. However, since $g_{nn} \geq 0$, g_n is either constant or strictly increasing and hence non-negative for all n greater than the smallest solution \tilde{n} . As a consequence, g is non-negative for all n greater than the smallest \tilde{n} . Hence, the nominator is either zero and the frontier horizontal or the nominator is positive and the frontier has a negative slope. The slope of the efficiency frontier is everywhere non-positive.

Let $g(0, \tilde{\delta}) = 0$. If $g_n(0, \tilde{\delta}) < 0$, $g(n, \tilde{\delta}) < 0$ for $0 < n < \tilde{n}$ with \tilde{n} the smallest non-zero solution to $g(n, \tilde{\delta}) = 0$. The argument above applies and $n = 0$ is not on the efficiency frontier. If $g_n(0, \tilde{\delta}) \geq 0$, $n = 0$ is on the efficiency frontier which is vertical.

C GENERAL RULE

Let the function r of the parameters n and δ serve as a criterion for the introduction of a planning measure. In case of the percentage criterion we can write r as:

$$r = \frac{\delta}{\Delta} \quad (\text{C.1})$$

and the policy is implemented if $r \geq r^*$, the criterion set by the planner. Similarly, the Finnish rule becomes:

$$r = (r_1, r_2) = \left(n, \frac{\delta}{\Delta}\right) \quad (\text{C.2})$$

and our “simple” rule:

$$r = (r_1, r_2) = (n, \delta) \quad (\text{C.3})$$

The policy is approved of if $r = (r_1, r_2) \geq (r_1^*, r_2^*) = r^*$. The value of r^* is chosen by the planner. We will call the vector r the (decision) rule and the value r^* , which r has to reach or exceed, the (decision) criterion.

D THE GENERAL STRUCTURE OF THE OPTIMAL POLICY PROBLEM

D.1 NOTATION AND DEFINITIONS

Jurisdictions are characterized by $(n, \delta) \in L \subset \mathbb{R}_+^2$. L is the set of relevant points in the $(n - \delta)$ -space. The distribution of the jurisdictions is described by a density measure $m(n, \delta)$.³¹ The planning measures are denoted by i with $i \in I$, I being the set of all planning measures considered, and the policy categories by b , $b \in B$, where B is the list of all possible combinations of one or more planning measures. The set of planning measures in category b is I^b .

A policy, indexed by $p \in P$, where P is the list of all possible policies, is denoted by \mathfrak{P}^p . It is a collection of categories b , each category being a collection of measures I^b . Following the definition in section 2.3, we note that for each policy p :

$$\bigcup_{I^b \in \mathfrak{P}^p} I^b = I \text{ and } I^b \cap I^{b'} = \emptyset \text{ for } b \neq b' \quad (\text{D.1})$$

The set of all possible policies is denoted by \mathbb{P} : $\forall p, \mathfrak{P}^p \in \mathbb{P}$.

The net benefit of planning measure i in a jurisdiction characterized by (n, δ) is:

$$u^i(n, \delta) := \beta^i n - c^i(n, \delta) \quad (\text{D.2})$$

A policy rule is a vector r and a critical value of the rule in category b is the vector r^b . The set of jurisdictions where the policy is implemented using this rule is called $J^b(r^b)$:

$$J^b(r^b) := \{(n, \delta) \in L \mid r \geq r^b\} \quad (\text{D.3})$$

³¹ In the discrete case, this measure degenerates to a sum of products of Dirac δ functions: $m = \sum_{k \in K} \delta(n - n^k) \delta(\delta - \delta^k)$. The jurisdictions in the discrete set of jurisdictions are numbered by k , $k \in K$, K being the set of all jurisdictions considered.

The set of jurisdictions, where the policy is not implemented, is then:

$$N^b(r^b) := \{(n, \delta) \in L \mid r < r^b\} \quad (\text{D.4})$$

The aggregated net benefits of a measure $i \in I^b$ under critical value r^b is then:

$$U^i(r^b) := \int_{J^b(r^b)} u^i(n, \delta) m(n, \delta) \, dn \, d\delta \quad (\text{D.5})$$

and the aggregated net benefit of the category is:

$$U^b(r^b) := \sum_{i \in I^b} \int_{J^b(r^b)} u^i(n, \delta) m(n, \delta) \, dn \, d\delta \quad (\text{D.6})$$

D.2 BENCHMARK

The unconstrained first-best optimum would imply the realization of all planning measures in each jurisdiction that give a positive net benefit in the respective jurisdiction. This gives us the bench mark for the aggregated net benefits:

$$U_{max} := \sum_{i \in I} \int_L \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \quad (\text{D.7})$$

Similarly, we find the benchmark for any category b :

$$U_{max}^b := \sum_{i \in I^b} \int_L \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \quad (\text{D.8})$$

D.3 CATEGORIES

The difference between net benefits of the benchmark and the net benefits realized under the policy rule is:

$$\begin{aligned} & U_{max}^b - U^b(r^b) \\ &= \sum_{i \in I^b} \int_L \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta - \sum_{i \in I^b} \int_{J^b(r^b)} u^i(n, \delta) m(n, \delta) \, dn \, d\delta \\ &= \sum_{i \in I^b} \left[\int_{J^b(r^b)} \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta + \int_{N^b(r^b)} \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \right. \\ &\quad \left. - \int_{J^b(r^b)} u^i(n, \delta) m(n, \delta) \, dn \, d\delta \right] \quad (\text{D.9}) \\ &= \sum_{i \in I^b} \left[\int_{N^b(r^b)} \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta - \int_{J^b(r^b)} \min \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \right] \\ &= \sum_{i \in I^b} \left[\int_{N^b(r^b)} \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta + \int_{J^b(r^b)} \max \{-u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \right] \end{aligned}$$

The two expressions in the last row within the square brackets of display D.9 are simply the exclusion errors – the net values of planning measure i in the different jurisdictions where it is not implemented although having a positive value and hence being efficient – as well as the inclusion errors – the net absolute value of measure i in jurisdictions where it is implemented although having a negative value and hence being inefficient:

$$E_e^i(r^b) := \int_{N^b(r^b)} \max \{u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \quad (\text{D.10})$$

and

$$E_i^i(r^b) := \int_{J^b(r^b)} \max \{-u^i(n, \delta), 0\} m(n, \delta) \, dn \, d\delta \quad (\text{D.11})$$

Aggregating the errors over all planning measures in category b , I^b , we arrive at the total errors of the two types in the chosen category:

$$E_e^b(r^b) := \sum_{i \in I^b} E_e^i(r^b) \quad (\text{D.12})$$

and

$$E_i^b(r^b) := \sum_{i \in I^b} E_i^i(r^b) \quad (\text{D.13})$$

Expression D.9 can now be reformulated as:

$$U^b(r^b) = U_{max}^b - [E_e^b(r^b) + E_i^b(r^b)] \quad (\text{D.14})$$

A planner maximizing net benefits should hence choose r^b in order to minimize the sum of errors.

Of course, the planner might want to support the minority beyond looking for efficiency. This could be incorporated into the analysis by using a different objective function. One such possibility is adding a term which expresses the planners desire to redistribute in favor of the minority, giving less weight to the inclusion error and more weight to the exclusion error. One possibility is:

$$\Delta E_{red}^b(r^b) := \alpha E_e^b(r^b) - \alpha E_i^b(r^b) \quad (\text{D.15})$$

with $0 \leq \alpha < 1$. That gives us:

$$U_{red}^b(r^b) := U_{max}^b - [E_e^b(r^b) + E_i^b(r^b) + \Delta E_{red}^b(r^b)] \quad (\text{D.16})$$

$$\begin{aligned} &= U_{max}^b - [(1 + \alpha)E_e^b(r^b) + (1 - \alpha)E_i^b(r^b)] \\ &=: U_{max}^b - E_{red}^b \end{aligned} \quad (\text{D.17})$$

The objective of the planner is to maximize U_{red}^b or, the same objective, minimize E_{red}^b for each category b . This is achieved by choosing an optimal values of r^b : r^{b*} . The corresponding error is E_{red}^{b*} .

D.4 POLICIES

In section D.3 we found the optimal rule (r^{b*}) and corresponding planning error E_{red}^{b*} in category b in the eyes of the planner. The planning error of policy p is then:

$$E(\mathfrak{P}^p) := \sum_{I^b \in \mathfrak{P}} E_{red}^{b*} \quad (D.18)$$

Let the administrative costs of policy p be $C_A(\mathfrak{P}^p)$. Presumably they are related to the number of categories in the policy, since the number of categories determines the number of bureaucratic processes necessary to implement the policy.

The social value of policy p is

$$W(\mathfrak{P}^p) := U_{max} - E(\mathfrak{P}^p) - C_A(\mathfrak{P}^p), \quad (D.19)$$

and the optimal policy is given by

$$\mathfrak{P}^* := \operatorname{argmax} \{W(\mathfrak{P}^p) \mid \mathfrak{P}^p \in \mathbb{P}\}. \quad (D.20)$$

For the sake of simplicity, we assume it to be unique. \mathfrak{P}^* now gives us the set of optimal categories,

$$B^* := \{b \in B \mid I^b \in \mathfrak{P}^*\} \quad (D.21)$$

as well as the optimal rules:

$$R^* := \{r^{b*} \mid b \in B^*\} \quad (D.22)$$

E OPTIMAL CRITERION FOR A GIVEN CATEGORY

Let the set $P^i := \{(n, \delta) \in L \mid u^i \geq 0\}$. That is, P^i is the set of jurisdictions in which planning measure i is efficient. Also define $R(r^b) := \{(n, \delta) \in L \mid r \geq r^b\}$. $R(r^b)$ is then the set of jurisdictions in which category b is implemented. The exclusion error due to planning measure i is now:

$$E_e^i(r^b) = \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \int_{R(r^b)} \max\{u^i(n, \delta), 0\} m(n, \delta) dnd\delta \quad (E.1)$$

and the inclusion error is:

$$E_i^i(r^b) = - \int_{R(r^b)} \min\{u^i(n, \delta), 0\} m(n, \delta) dnd\delta \quad (E.2)$$

Hence, the total planning error due to planning measure i is:³²

$$E^i(r^b) = \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \int_{R(r^b)} u^i(n, \delta) m(n, \delta) dnd\delta \quad (E.3)$$

³² We are from now on assuming that the planner is only interested in efficiency. Consequently $\alpha = 0$ in expression D.15 and E_{red} is written E .

The planning error in category b is then:

$$E^b(r^b) = \sum_{i \in I^b} E^i = \sum_{i \in I^b} \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \int_{R(r^b)} \sum_{i \in I^b} u^i(n, \delta) m(n, \delta) dnd\delta \quad (\text{E.4})$$

We want to choose the planning rule r^b in order to minimize E^b , that is, in order to maximize the second term, written E_2^b , since the first term does not depend on the rule. The second term can easily be written explicitly if the rule $r = (n, \delta)$:

$$\begin{aligned} E_2^b(n^b, \delta^b) &:= \int_{R(n^b, \delta^b)} \sum_{i \in I^b} u^i(n, \delta) m(n, \delta) dnd\delta \\ &= \int_{\delta^b}^{\infty} \int_{n^b}^{\infty} \sum_{i \in I^b} [\beta^i n - c^i(n, \delta)] m(n, \delta) dnd\delta \\ &= \int_{\delta^b}^{\infty} \int_{n^b}^{\infty} \sum_{i \in I^b} \beta^i \left[n - \frac{c^i(n, \delta)}{\beta^i} \right] m(n, \delta) dnd\delta \\ &= \int_{\delta^b}^{\infty} \int_{n^b}^{\infty} \sum_{i \in I^b} \nu^b \bar{\beta}^b \gamma^i \left[n - \frac{c^i(n, \delta)}{\beta^i} \right] m(n, \delta) dnd\delta \\ &= \nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} \int_{n^b}^{\infty} \left[n - \sum_{i \in I^b} \gamma^i \frac{c^i(n, \delta)}{\beta^i} \right] m(n, \delta) dnd\delta \end{aligned} \quad (\text{E.5})$$

We have denoted the number of measures in category b by ν^b and defined $\bar{\beta}^b$ by:

$$\bar{\beta}^b := \frac{\sum_{i \in I^b} \beta^i}{\nu^b} \quad (\text{E.6})$$

Taking the derivatives with respect to n^b and δ^b , we find:

$$\begin{aligned} \frac{\partial E_2^b}{\partial n^b} &= -\nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} \left[n^b - \sum_{i \in I^b} \gamma^i \frac{c^i(n^b, \delta)}{\beta^i} \right] m(n^b, \delta) d\delta \\ &= -\nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} m(n^b, \delta) d\delta \left[n^b - \int_{\delta^b}^{\infty} w(n^b, \delta^b; \delta) \sum_{i \in I^b} \gamma^i \frac{c^i(n^b, \delta)}{\beta^i} d\delta \right] \\ &= -\nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} m(n^b, \delta) d\delta \left[n^b - \sum_{i \in I^b} \gamma^i \int_{\delta^b}^{\infty} w(n^b, \delta^b; \delta) \frac{c^i(n^b, \delta)}{\beta^i} d\delta \right] \\ &= -\nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} m(n^b, \delta) d\delta \left[n^b - \sum_{i \in I^b} \gamma^i \int_{\delta^b}^{\infty} \tilde{n}^i(n^b, \delta) w(n^b, \delta^b; \delta) d\delta \right] \\ &= -\nu^b \bar{\beta}^b \int_{\delta^b}^{\infty} m(n^b, \delta) d\delta \left[n^b - \sum_{i \in I^b} \gamma^i \tilde{n}^i(n^b, \delta^b) \right] \end{aligned} \quad (\text{E.7})$$

and

$$\begin{aligned}
\frac{\partial E_2^b}{\partial \delta^b} &= -v^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^b) \left[n - \sum_{i \in I^b} \gamma^i \frac{c^i(n, \delta^b)}{\beta^i} \right] dn \\
&= -v^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^b) \frac{n}{\delta^b} \left[\delta^b - \sum_{i \in I^b} \gamma^i \frac{\delta^b}{n} \frac{c^i(n, \delta^b)}{\beta^i} \right] dn \\
&= -v^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^b) \frac{n}{\delta^b} \left[\delta^b - \sum_{i \in I^b} \gamma^i \check{\delta}^i(n, \delta^b) \right] dn \tag{E.8} \\
&= -v^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^b) \frac{n}{\delta^b} dn \left[\delta^b - \sum_{i \in I^b} \gamma^i \int_{n^b}^{\infty} \omega(n^b, \delta^b; n) \check{\delta}^i(n, \delta^b) dn \right] \\
&= -v^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^b) \frac{n}{\delta^b} dn \left[\delta^b - \sum_{i \in I^b} \gamma^i \tilde{\delta}^i(n^b, \delta^b) \right]
\end{aligned}$$

The weights γ^i are defined as:

$$\gamma^i := \frac{\beta^i}{\sum_{i \in I^b} \beta^i} \tag{E.9}$$

We have also defined:

$$\check{n}^i(n^b, \delta) := \frac{c^i(n^b, \delta)}{\beta^i} \tag{E.10}$$

and

$$\check{\delta}^i(n, \delta^b) := \frac{\delta^b}{n} \frac{c^i(n, \delta^b)}{\beta^i} \tag{E.11}$$

Writing the efficiency frontier as $n^i(\delta)$ or $\delta^i(n)$, respectively, and assuming that they are well-defined³³ we can use the definition $\beta^i n^i(\delta) = c^i(n^i(\delta), \delta)$ or $\beta^i n = c^i(n, \delta^i(n))$ and rewrite these variables as:

$$\check{n}^i(n^b, \delta) = n^i(\delta) \frac{c^i(n^b, \delta)}{c^i(n^i(\delta), \delta)} \tag{E.12}$$

and

$$\check{\delta}^i(n, \delta^b) = \delta^b \frac{c^i(n, \delta^b)}{c^i(n, \delta^i(n))} \tag{E.13}$$

³³ In the case of a vertical efficiency frontier $\delta^i(n)$ is not well-defined and in the case of a horizontal efficiency frontier $n^i(\delta) = 0$ for $\delta \geq \delta^*$.

The weighted averages \tilde{n}^i and $\tilde{\delta}^i$ are defined as:

$$\tilde{n}^i(n^b, \delta^b) := \int_{\delta^b}^{\infty} \tilde{n}^i(n^b, \delta) w(n^b, \delta^b; \delta) d\delta \quad (\text{E.14})$$

and

$$\tilde{\delta}^i(n^b, \delta^b) := \int_{n^b}^{\infty} \tilde{\delta}^i(n, \delta^b) \omega(n^b, \delta^b; n) dn \quad (\text{E.15})$$

with the weights w and ω given by:

$$w(n^b, \delta^b; \delta) := \frac{m(n^b, \delta)}{\int_{\delta^b}^{\infty} m(n^b, \delta) d\delta}, \quad \int_{\delta^b}^{\infty} w(n^b, \delta^b; \delta) d\delta = 1 \quad (\text{E.16})$$

and

$$\omega(n^b, \delta^b; n) := \frac{m(n, \delta^b)n}{\int_{n^b}^{\infty} m(n, \delta^b)n dn}, \quad \int_{n^b}^{\infty} \omega(n^b, \delta^b; n) dn = 1 \quad (\text{E.17})$$

If the costs are constant the efficiency frontier n^i is also constant and $\tilde{n}^i(n^b, \delta^b) = n^i$. Also if the costs do not depend on the area, the efficiency frontier is constant and $\tilde{n}^i(n^b, \delta^b) = n^i c^i(n^b, \cdot) / c^i(n^i, \cdot) = c^i(n^b, \cdot) / \beta^i$. If the costs are independent of the number of beneficiaries and proportional to the area of application, the efficiency frontier δ^i is constant and $\tilde{\delta}^i(n^b, \delta^b) = \delta^i$. Intuitively, $\tilde{n}^i(n^b, \delta)$ can be seen as the efficiency frontier corrected for costs; in the case of exclusion errors it is bigger than or equal to $n^i(\delta)$ and hence in general closer to n^b than $n^i(\delta)$ and in the case of inclusion errors it is smaller than or equal to $n^i(\delta)$ and hence in general closer to n^b than $n^i(\delta)$.³⁴ That is, it is in general closer to n^b than $n^i(\delta)$. $\tilde{n}^i(n^b, \delta^b)$ is then found by averaging $\tilde{n}^i(n^b, \delta)$ over all $\delta \geq \delta^b$ with the weights determined by the density of jurisdictions. Similarly, $\tilde{\delta}^i(n)$ is closer to δ^b than $\delta^i(n)$ and averaged for $n \geq n^b$.

An inner solution to the maximization problem – a positive critical value for both n and δ – gives an optimal rule characterized by:

$$n^{b*} = \sum_{i \in I^b} \gamma^i \tilde{n}^i(n^{b*}, \delta^{b*}) \quad (\text{E.18})$$

and

$$\delta^{b*} = \sum_{i \in I^b} \gamma^i \tilde{\delta}^i(n^{b*}, \delta^{b*}) \quad (\text{E.19})$$

We have, hence, reduced the structure of the optimal critical values of the (n, δ) -rule in a category to the determination of two factors: the importance of the measures given by the γ s and the structure of the costs of the measures given by the \tilde{n} s and $\tilde{\delta}$ s. That is, the optimal n is given as a weighted average of the modified efficiency frontiers with the weights being the β^i s, that

³⁴ Equality holds if the costs are independent of the number of beneficiaries.

is the importance of the measures.³⁵ If the costs are independent of the population density the efficiency frontier is vertical and the expressions are reduced to:

$$n^{b*} = \sum_{i \in I^b} \gamma^i \frac{c^i(n^{b*}, \cdot)}{\beta^i} = \sum_{i \in I^b} \gamma^i n^i(\cdot) \frac{c^i(n^{b*}, \cdot)}{c^i(n^i(\cdot), \cdot)} \quad (\text{E.20})$$

and

$$\frac{\partial E_2^b}{\partial \delta^b} = -\nu^b \bar{\beta}^b \int_{n^b}^{\infty} m(n, \delta^{b*}) \left[n - \sum_{i \in I^b} \gamma^i \frac{c^i(n, \cdot)}{\beta^i} \right] dn \quad (\text{E.21})$$

If equation E.20 characterizes an optimal rule, then $n > \sum_{i \in I^b} \gamma^i c^i(n, \cdot) / \beta^i$ for all $n > n^{b*}$, and the expression for $\partial E_2^b / \partial \delta^b$ will be negative implying that the optimal $\delta^{b*} = 0$.

In the case of non-rival and non-spatial cost structures the costs are independent of n , too and the optimal n , as we have seen, is further reduced to:

$$n^{b*} = \sum_{i \in I^b} \gamma^i n^i \quad (\text{E.22})$$

F THE SIZE OF THE TOTAL PLANNING ERROR IN A GIVEN CATEGORY

In order to estimate the approximate error of a category b under rule $r = (n, \delta)$ it is convenient to divide the $(n - \delta)$ -diagram into three regions. We first define two parameters \hat{n}^i and $\hat{\delta}^i$:

$$(\hat{n}^i, \hat{\delta}^i) := \{ (n, \delta) \geq (n^*, \delta^*) \mid \beta^i n - c^i(n, \delta) = 0 \} \wedge n^* - n = \delta^* - \delta \} \quad (\text{F.1})$$

or, if this set is empty, $(\hat{n}^i, \hat{\delta}^i) := (n^*, \delta^*)$. In view of proposition 1 the pair $(\hat{n}^i, \hat{\delta}^i)$ is unique. By construction it can also be ordered by the vector relation \geq . Let $(\hat{n}^d, \hat{\delta}^d)$ be the biggest pair under this relation when $i \in I^d$. The choice of $(\hat{n}^i, \hat{\delta}^i)$ is illustrated in figure F.1.

With the help of $(\hat{n}^d, \hat{\delta}^d)$ the relevant part of the (n, δ) -space can be divided into three areas. This is illustrated in figure F.2. In order to estimate the errors, one can integrate over the various areas separately. For area A it is sensible to first integrate $u^i(n, \delta) m(n, \delta)$ over n for a given value of δ from the efficiency frontier to n^* .³⁶ Integrating the result of the first integral from $\hat{\delta}^d$ to infinity will then give the total error in area A .

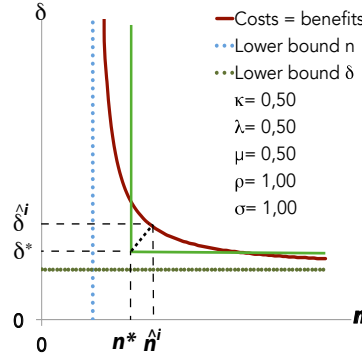
For area B we can proceed in the same manner, only first integrating over δ and then over n from \hat{n}^d to infinity. For area C it is convenient to change to polar coordinates (ρ, ψ) anchored in $(\hat{n}^d, \hat{\delta}^d)$, first integrating over the radius from the efficiency frontier to the value of ρ defined by the rule:

$$\rho^*(\psi) := \min \left\{ \frac{\hat{n}^d - n^*}{\sin(\psi)}, \frac{\hat{\delta}^d - \delta^*}{\cos(\psi)} \right\} \quad (\text{F.2})$$

and then over the angle ψ from zero to $\pi/2$.

³⁵ The expressions characterize an optimum. In order to guarantee its uniqueness we would have to impose conditions on the density functions m . For the sake of brevity of the presentation we ignore this problem here.

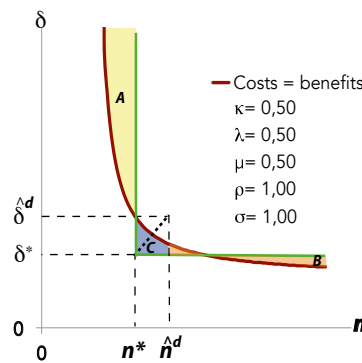
³⁶ Note that u^i is positive to the right of the frontier and negative to the left of it. Hence, integrating from the frontier to n^* automatically makes the errors of both kinds positive.

FIGURE F.1 Definition of $(\hat{n}^i, \hat{\delta}^i)$.

We will limit the analysis to values of δ above $\hat{\delta}^b$ (region A). An analysis of the errors for other areas of the (n, δ) -space is, as we have suggested, analogous and leads to similar results. Let $n^i(\delta)$ be the efficiency frontier for $\delta \geq \hat{\delta}^b$. The planning errors due to policy i for $\delta \geq \hat{\delta}^b$ are then given by:

$$\begin{aligned}
 E_{\delta \geq \hat{\delta}^b}^i &:= \int_{\hat{\delta}^b}^{\infty} \int_{n^i(\delta)}^{n^{b*}} [\beta^i n - c^i(n, \delta)] m(n, \delta) dn d\delta \\
 &= \beta^i \int_{\hat{\delta}^b}^{\infty} \int_{n^i(\delta)}^{n^{b*}} \left[n - \frac{c^i(n, \delta)}{\beta^i} \right] m(n, \delta) dn d\delta
 \end{aligned} \tag{F.3}$$

We note that the expression $n - c^i(n, \delta)/\beta^i$ in the inner integral is a convex function of n if $n^{b*} \geq n^i(\delta)$ and that $c^i(n, \delta)/\beta^i - n$ is a concave function of n if $n^{b*} \leq n^i(\delta)$. In order to get a lower bound for the integral under the assumption that $m(n, \delta)$ is independent of n between the efficiency frontier and the optimal criterion, we can linearize the argument for $n^{b*} \leq n^i(\delta)$. If $n^{b*} \geq n^i(\delta)$ the integral is bigger than a corresponding integral of $c^i(n, \delta)/\beta^i - n$ from $n^i(\delta) - (n^{b*} - n^i(\delta))$ to $n^i(\delta)$ due to the concavity of the costs in n . Substituting this integral for the case that $n^{b*} \geq n^i(\delta)$ and linearizing gives us a lower bound for the size of the errors in

FIGURE F.2 Definition of three areas of the (n, δ) -space.

area A :

$$\begin{aligned}
E_{\delta \geq \hat{\delta}^b}^i &\geq \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} (n^i(\delta) - n^{b*}) \left(\frac{c^i(n^{b*}, \delta)}{\beta^i} - n^{b*} \right) m(n^{b*}, \delta) d\delta \\
&= \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} (n^i(\delta) - n^{b*}) \left(\frac{c^i(n^{b*}, \delta)}{c^i(n^i(\delta), \delta)} n^i(\delta) - n^{b*} \right) m(n^{b*}, \delta) d\delta \\
&\geq \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} m(n^{b*}, \delta) d\delta \int_{\hat{\delta}^b}^{\infty} \left(\frac{c^i(n^{b*}, \delta)}{c^i(n^i(\delta), \delta)} n^i(\delta) - n^{b*} \right)^2 w(n^{b*}, \hat{\delta}^b; \delta) d\delta \\
&= v^b \frac{\bar{\beta}}{2} \int_{\hat{\delta}^b}^{\infty} m(n^{b*}, \delta) d\delta \gamma^i \int_{\hat{\delta}^b}^{\infty} (\check{n}^i(n^{b*}, \delta) - n^{b*})^2 w(n^{b*}, \hat{\delta}^b; \delta) d\delta
\end{aligned} \tag{F.4}$$

if $n^{b*} \leq n^i(\delta)$ and, defining $n_M^{b*} := 2n^i(\delta) - n^{b*}$,

$$\begin{aligned}
E_{\delta \geq \hat{\delta}^b}^i &\geq \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} (n^i(\delta) - n_M^{b*}(\delta)) \left(\frac{c^i(n_M^{b*}(\delta), \delta)}{\beta^i} - n_M^{b*}(\delta) \right) m(n_M^{b*}(\delta), \delta) d\delta \\
&= \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} (n^i(\delta) - n_M^{b*}(\delta)) \left(\frac{c^i(n_M^{b*}(\delta), \delta)}{c^i(n^i(\delta), \delta)} n^i(\delta) - n_M^{b*}(\delta) \right) m(n^{b*}, \delta) d\delta \\
&\geq \frac{\beta^i}{2} \int_{\hat{\delta}^b}^{\infty} \left(\frac{c^i(n_M^{b*}(\delta), \delta)}{c^i(n^i(\delta), \delta)} n^i(\delta) - n_M^{b*}(\delta) \right)^2 m(n^{b*}, \delta) d\delta \\
&= v^b \frac{\bar{\beta}}{2} \int_{\hat{\delta}^b}^{\infty} m(n^{b*}, \delta) d\delta \gamma^i \int_{\hat{\delta}^b}^{\infty} (\check{n}^i(n_M^{b*}(\delta), \delta) - n_M^{b*}(\delta))^2 w(n^{b*}, \hat{\delta}^b; \delta) d\delta
\end{aligned} \tag{F.5}$$

if $n^{b*} \geq n^i(\delta)$. We have . We remind the reader that $n^i(\delta) - n_M^{b*}(\delta) = n^{b*} - n^i(\delta)$; $n_M^{b*}(\delta)$ and n^{b*} are equidistant from $n^i(\delta)$ by construction; $n_M^{b*}(\delta)$ is the mirror image of n^{b*} . It is instructive to calculate the difference between $\check{n}^i(n_M^{b*}(\delta), \delta) - n_M^{b*}(\delta)$ and $n^{b*} - \check{n}^i(n^{b*}, \delta)$:

$$n^{b*} - \check{n}^i(n^{b*}, \delta) - \check{n}^i(n_M^{b*}(\delta), \delta) + n_M^{b*}(\delta) = 2n^i(\delta) \left(1 - \frac{1}{2} \frac{c^i(n^{b*}, \delta) + c^i(n_M^{b*}(\delta), \delta)}{c^i(n^i(\delta))} \right) \geq 0 \tag{F.6}$$

The difference increases with the concavity of the cost function and if the costs are linear in n the difference vanishes. If the cost function is linear in n also the expression for the errors becomes exact, see below.

Taking the sum over all planning measures in b we arrive at:

$$E_{\delta \geq \hat{\delta}^{b*}}^b \geq v^b \frac{\bar{\beta}}{2} \int_{\hat{\delta}^b}^{\infty} m(n^{b*}, \delta) d\delta \sigma_b^2 \tag{F.7}$$

The generalized weighted variance σ_b^2 is defined as:

$$\begin{aligned}
\sigma_b^2 &:= \sum_{i \in I^b} \gamma^i \int_{\hat{\delta}^b}^{\infty} w(n^{b*}, \hat{\delta}^b; \delta) \left[\varepsilon(d) (\check{n}^i(n^{b*}, \delta) - n^{b*})^2 \right. \\
&\quad \left. + [1 - \varepsilon(d)] (\check{n}_M^i(n^{b*}, \delta) - n_M^{b*})^2 \right] d\delta
\end{aligned} \tag{F.8}$$

Where d is defined by $d := n^i(\delta) - n^{b*}$ and the function $\varepsilon(d)$ takes the value 0 for $d < 0$ and the value 1 for $d \geq 0$.

The variance is basically a measure of how close the efficiency frontiers of different planning measures are after we have corrected them for the concavity of the cost functions. The error of an individual measure increases, on the one hand, with the square of the difference between the efficiency frontier – averaged over δ and corrected for the concavity of the cost function – and the line determined by the optimal criterion for n and, on the other hand, the importance of the measure given by the benefit β^i as well as the density of the jurisdictions in the area between the efficiency frontier and the criterion n^* , area A . That is, for a given distribution of the jurisdictions on n and δ two factors contribute to the planning errors, the importance of the measures and differences in their cost structures.

As an illustration – and exact calculation – consider the case of costs dependent neither on n nor on δ (the non-rival, non-spatial case) and the distribution of jurisdictions being constant with respect to n in the relevant area ($m(n, \delta) = m(n', \delta) = m(n^{b*}, \delta) \forall n, n' \in [n_{min}^i, n_{max}^i], i \in I^b$):

$$\begin{aligned}
 E_{\delta \geq 0}^i &= \int_0^\infty \int_{n^i}^{n^{b*}} [\beta^i n - c^i(\cdot, \cdot)] m(n^{b*}, \delta) dn d\delta \\
 &= \beta^i \int_0^\infty m(n^{b*}, \delta) d\delta \int_{n^i}^{n^{b*}} \left[n - \frac{c^i(\cdot, \cdot)}{\beta^i} \right] dn \\
 &= \beta^i \int_0^\infty m(n^{b*}, \delta) d\delta \int_{n^i}^{n^{b*}} [n - n^i] dn \\
 &= \beta^i \int_0^\infty m(n^{b*}, \delta) d\delta \left[\frac{n^{b*2}}{2} - \frac{n^i2}{2} - n^i n^{b*} + n^i2 \right] \\
 &= \frac{\beta^i}{2} \int_0^\infty m(n^{b*}, \delta) d\delta [n^{b*} - n^i]^2
 \end{aligned} \tag{F.9}$$

Taking the sum over all planning measures in category b we finally get:

$$\begin{aligned}
 E_{\delta \geq 0}^b &= \sum_{i \in I^b} E_{\delta \geq 0}^i = \sum_{i \in I^b} \frac{\beta^i}{2} \int_0^\infty m(n^{b*}, \delta) d\delta [n^{b*} - n^i]^2 \\
 &= v^b \frac{\bar{\beta}^b}{2} \int_0^\infty m(n^{b*}, \delta) d\delta \sum_{i \in I^b} \gamma^i [n^{b*} - n^i]^2 \\
 &= \frac{v^b \bar{\beta}^b}{2} \int_0^\infty m(n^{b*}, \delta) d\delta \sigma_b^2
 \end{aligned} \tag{F.10}$$

The weighted variance becomes:

$$\sigma_b^2 = \sum_{i \in I^b} \gamma^i [n^{b*} - n^i]^2 \tag{F.11}$$

G PROOF THAT TOTAL PLANNING COSTS DECREASE AS CATEGORIES ARE DIVIDED

In equation E.4 the planning error is separated into one part that is independent of the planning rule and one part depending on it. In the second term we can reverse the order of the sum and the integral. We partition the set I^b in two sets, I^{b_1} and I^{b_2} :

$$I^b = I^{b_1} \cup I^{b_2}, \quad I^{b_1} \cap I^{b_2} = \emptyset \quad (\text{G.1})$$

Equation E.4 can now be written as:

$$\begin{aligned} E^{b*} &= \sum_{i \in I^b} \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \sum_{i \in I^b} \int_{R(r^{b*})} u^i(n, \delta) m(n, \delta) dnd\delta \\ &= \sum_{i \in I^{b_1}} \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \sum_{i \in I^{b_1}} \int_{R(r^{b*})} u^i(n, \delta) m(n, \delta) dnd\delta \\ &\quad + \sum_{i \in I^{b_2}} \int_{P^i} u^i(n, \delta) m(n, \delta) dnd\delta - \sum_{i \in I^{b_2}} \int_{R(r^{b*})} u^i(n, \delta) m(n, \delta) dnd\delta \\ &= E^{b_1}(r^{b*}) + E^{b_2}(r^{b*}) \end{aligned} \quad (\text{G.2})$$

Let r^{b_1*} and r^{b_2*} be the policies that minimize the size of the errors $E^{b_1}(r^b)$ and $E^{b_2}(r^b)$ respectively:

$$E^{b_1*} := E^{b_1}(r^{b_1*}) \leq E^{b_1}(r^{b*}) \quad (\text{G.3})$$

$$E^{b_2*} := E^{b_2}(r^{b_2*}) \leq E^{b_2}(r^{b*}) \quad (\text{G.4})$$

The inequalities follow directly from the fact that the new values of the policy variables minimize the respective error sizes. Consequently:

$$E^{b*} \geq E^{b_1*} + E^{b_2*} \quad (\text{G.5})$$

It is readily seen that if the measures in the new categories are selected in order to make the new categories more homogeneous than the old category the inequalities will become strict. One could for instance choose the measures in the big category with the highest ratio of inclusion error to exclusion error for one of the new category and inversely for the other new category.

H THE OPTIMAL DISTRIBUTION OF MEASURES ON CATEGORIES

When a measure is added to a category the size of the error is affected in two ways. The measure itself brings a direct error dependent on the criterion r^b and the optimal criterion is changed due to the new measure. Both changes in the size of the errors will depend on the importance of the measure (β^i) and the cost structure, captured in the efficiency frontier.

Assume that a rule r is applied to two categories, b_1 and b_2 , with corresponding errors $E^1(r^1)$ and $E^2(r^2)$ for the criteria r^1 and r^2 . A new measure i is introduced and should be added to one of the categories. The error due to measure i for criterion r^i is $E^i(r^i)$. If the

measure is added to category b_1 the optimal criterion changes from r^{1*} to r^{1**} and if it is added to b_2 it changes from r^{2*} to r^{2**} ; we define $\Delta r^1 := r^{1**} - r^{1*}$ and $\Delta r^2 := r^{2**} - r^{2*}$.

The change in the size of the error in category b_1 if the measure is added is then:

$$\begin{aligned}\Delta E^{b_1} &= E^1(r^{1**}) - E^1(r^{1*}) + E^i(r^{1**}) - E^i(r^{1*}) + E^i(r^{1*}) \\ &= \Delta E^1(\Delta r^1) + \Delta E^i(\Delta r^1) + E^i(r^{1*})\end{aligned}\quad (\text{H.1})$$

with $\Delta E^1(\Delta r^1) := E^1(r^{1**}) - E^1(r^{1*})$ and $\Delta E^i(\Delta r^1) := E^i(r^{1**}) - E^i(r^{1*})$. For category b_2 the corresponding expression is:

$$\begin{aligned}\Delta E^{b_2} &= E^2(r^{2**}) - E^2(r^{2*}) + E^i(r^{2**}) - E^i(r^{2*}) + E^i(r^{2*}) \\ &= \Delta E^2(\Delta r^2) + \Delta E^i(\Delta r^2) + E^i(r^{2*})\end{aligned}\quad (\text{H.2})$$

It is optimal to add measure i to category b_2 if $\Delta E^{b_1} \geq \Delta E^{b_2}$, that is if:³⁷

$$E^i(r^{1*}) - E^i(r^{2*}) \geq [\Delta E^2(\Delta r^2) + \Delta E^i(\Delta r^2)] - [\Delta E^1(\Delta r^1) + \Delta E^i(\Delta r^1)] \quad (\text{H.3})$$

If $r^{i*} = r^{2*}$, $r^{2**} = r^{2*}$ and condition H.3 becomes:

$$E^i(r^{1*}) + \Delta E^1(\Delta r^1) + \Delta E^i(\Delta r^1) \geq E^i(r^{i*}) \quad (\text{H.4})$$

This inequality is always satisfied since $E^i(r^{i*})$ is the minimal error of measure i under rule r and hence the smallest possible increase in any category to which the measure is added, whereas the left-hand side is the increase in the category error if the measure were added to b^1 . We can conclude that if there is a category with the same optimal criterion as the optimal criterion of a given measure it is always optimal to add the measure to this criterion. By extension, if there are two categories with the same optimal criteria nothing is lost by combining them into one category. On the other hand, there could be gains in reallocating the measures into two new categories. In general, the closer is r^{i*} to r^{2*} , the smaller will be $E^i(r^{2*})$ and the more likely the satisfaction of inequality H.3.

We note that since r^{1*} and r^{2*} minimize the respective errors of categories b_1 and b_2 , small changes in the r s will have only a second-order effect on E^1 and E^2 , but a first order effect on E^i unless r^{i*} , the optimal criterion for measure i , is close to r^{1*} or r^{2*} .³⁸ We also note that the ΔE^i s are non-positive.³⁹ The Δr^i s will, on the one hand, depend on how close r^{i*} is to r^{1*} and r^{2*} , respectively, and, on the other hand, on the relative importance of measure i in comparison to the two categories. That is, for a given r^{i*} the smaller is $\sum_{i \in I^{b_2}} \beta^i$ in comparison to $\sum_{i \in I^{b_1}} \beta^i$, the bigger in absolute values is Δr^{2*} in comparison to Δr^{1*} and as a consequence the bigger in absolute value is $\Delta E^i(\Delta r^{2*})$ in comparison to $\Delta E^i(\Delta r^{1*})$. Since they are negative and (in absolute value) an order of magnitude larger than $\Delta E^1(\Delta r^{1*})$ and $\Delta E^2(\Delta r^{2*})$ the more likely is inequality H.3 satisfied. In the extreme case of $\sum_{i \in I^{b_2}} \beta^i = 0$, $r^{2**} = r^{i*}$ and the minimal error is added when i is added to b_2 . The result that a measure should be added to the category with the closest cost structure (close r^* s) has to be augmented with the result that this rule can be reversed if the categories are very different in importance (the sum of the β s), in which case the measure should be added to the category of least importance. There is a trade-off between closeness in cost structure of the new measure and one category and the difference in importance of the new measure and the other category.

³⁷ The inequalities can be made strict if the various minima of the errors are unique. It is straightforward to extend the analysis to include this case, albeit somewhat onerous.

³⁸ This is just a straight-forward application of the “envelope” theorem.

³⁹ Cf. proposition 7.